Optimal and adaptive planning of geotechnical construction: Preloading of a road embankment

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Abstract

To optimally design a geotechnical engineering structure, an iterative decision-making process is required due to the prevailing uncertainty of the ground conditions. In this paper, the authors show that such sequential decision making processes can be analyzed and optimized quantitatively, exemplifying with the design of a surcharge for an embankment on soft soil. The paper proposes a risk-based decision-theoretic approach to finding the optimal preloading sequence, i.e. finding the optimal surcharge height and, when needed, adapting it to the observed settlement. Adopting heuristics – a parametric description of preloading strategies – the approach balances the cost of surcharge material against financial penalties related to project delays and insufficient overconsolidation, which causes damage due to creep. The result is a preloading strategy that optimally accounts for information obtained from planned settlement measurements. The preloading planning problem is solved for different decision settings, going from optimizing a constant surcharge height, to finding the optimal time for adjusting the surcharge. The findings highlight the potential of using risk-based decision planning in geotechnical engineering, in particular in combination with the observational method.

Keywords: embankment, preloading, sequential decision problem, observational method

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1. Introduction

Design of geotechnical engineering structures implies decision making under uncertainty. The reason is mainly a lack of knowledge about the prevailing ground conditions, but there are also limitations in understanding and predicting the ground–structure interaction or temporal variations. Managing these uncertainties is essential to achieving a design of satisfactory quality without unnecessary delays and at a reasonable cost. One approach to this challenge is to view the geotechnical design and execution as a sequential decision problem, which has been studied in other areas of engineering and decision making (e.g., Rosenstein & Barto, 2001; Memarzadeh et al., 2014; Malings & Pozzi, 2016; Papakonstantinou & Shinozuka, 2014; Bismut & Straub, 2021; Wang et al., 2022). The aim is to find the sequence of decisions that minimize the expected design and construction costs. In the ideal case, the analysis should also consider operational and maintenance costs (Mendoza et al., 2021).

A typical example of a geotechnical engineer's decision under uncertainty is the design of embankments on soft soil prone to consolidation settlements. The embankment load initiates a consolidation process toward a final long-term settlement, but neither the magnitude of this settlement, nor the time until it is reached, can be well predicted by the engineer; despite geotechnical pre-investigations being performed, there are typically considerable uncertainties regarding the soil's hydraulic conductivity and deformation properties. Unless this uncertainty is carefully managed by a planned sequence of inspection decisions and mitigating actions during design and construction, unwanted costly consequences such as time delays or residual settlements after completion of the superstructure may occur. The engineering challenge therefore essentially lies in finding a cost-effective design solution, considering not only the technical requirements at the time of project completion, but also the respective probabilities and costs of potential consequences caused by an unsuccessful design.

One design alternative is to accelerate the consolidation by installing prefabricated vertical drains (PVDs) and preloading the embankment with a surcharge load (Figure 1) (Hansbo, 1979; Alonso et al., 2000; Walker & Indraratna, 2007; Indraratna et al., 2016; Geng & Yu, 2017; Guo et al., 2018; Nguyen et al., 2021). If a large enough surcharge load is used for a sufficiently long time and unloaded correctly, project delay and residual settlements can be avoided.

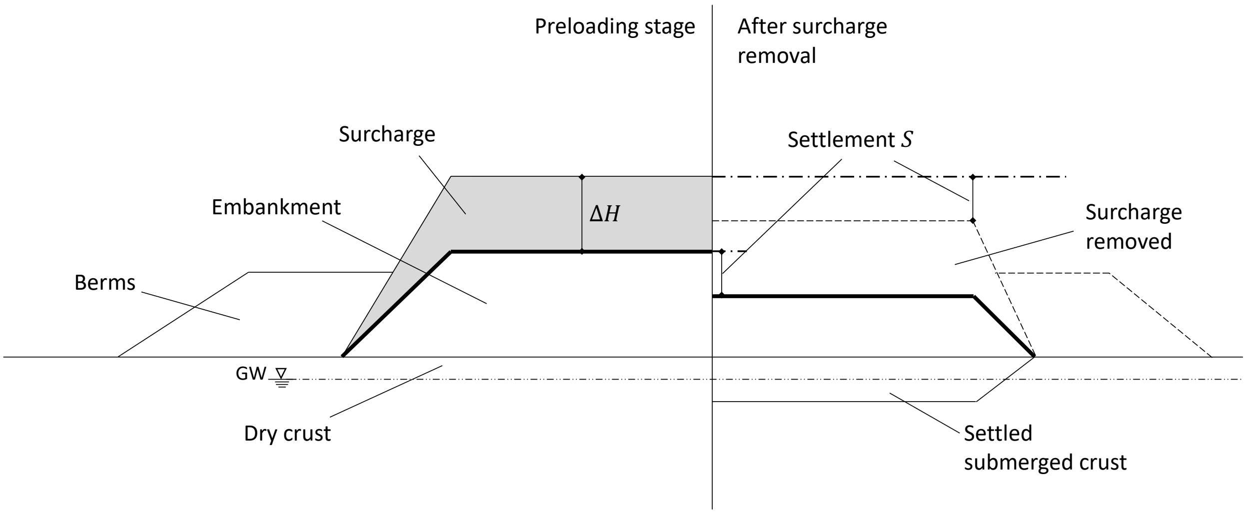


Figure 1: Preloading of an embankment with a surcharge of total height ΔH to accelerate consolidation. (GW: ground water).

The cost of surcharge material can be high, e.g., due to limited availability. This entices the engineer to optimize the cost of the surcharge against the risk of insufficient preloading. The engineer can also consider increasing the height of the surcharge at a later time, in response to observations of too slow settlement rates. However, this combination of having both sequential decisions on the applied surcharge height and prevailing uncertainties in the outcomes makes this a challenging optimization problem.

To the authors' knowledge, this problem – nor any other geotechnical problem – has never been formalized as a sequential decision problem. A few studies have however used other, simpler decision theoretical analyses for other geotechnical applications: Einstein et al. (1978) showed an early application of decision theoretical principles; Zetterlund et al. (2011), Sousa et al. (2017), and Klerk et al. (2019) performed value of information analyses; and preposterior analyses were performed by Schweckendiek & Vrouwenvelder (2013), Spross & Johansson (2017), van der Krogt et al. (2022), Löfman & Korkiala-Tanttu (2022), and Spross et al. (2022).

Probabilistic settlement analyses have recently been performed by e.g. Bari et al. (2016), Bong & Stuedlein (2018), and Löfman & Korkiala-Tanttu (2021). Addressing the design issue of embankment preloading with PVDs, Spross & Larsson (2021) specifically showed how a probabilistically evaluated initial surcharge height can be used in an observational method to limit the probability of time delay and residual settlement in soft soil. Spross et al. (2019) discussed how settlement monitoring can be evaluated as a basis for a decision to increase the surcharge height. The specific decision-theoretical problem was highlighted, but not solved.

In this paper, we propose a risk-based decision-theoretic approach to optimize the sequential decisions involved in embankment preloading. The sequence of decisions on initial surcharge height and later additions to the surcharge are optimized such that a desired settlement is achieved at a minimal expected cost, which reflects whether the settlement is achieved within a fixed time-frame. Construction delays as well as insufficient overconsolidation, which is a cause of residual settlement, are explicitly penalized.

We use Spross & Larsson (2021)'s probabilistic preloading model to describe the settlement evolution. This model in based on Hansbo (1979)'s analytical PVD model and Larsson & Sällfors (1986)'s analytical settlement model, in which the soil deformation properties are evaluated in constant-rate-of-strain tests. The probabilistic modelling of the soil properties is based on concepts developed by Phoon & Kulhawy (1999) and Müller et al. (2014, 2016). We extended the preloading model to allow simulation of soil settlement curves when the surcharge height is adjusted, thereby enabling modelling of the effect of sequential surcharge height decisions on the settlement evolution.

The outcome of the analysis is a preloading strategy, which prescribes how much surcharge to add conditional on settlement measurements. To tackle the added complexity of the optimization that arises from including these measurements in the decision process, we adopt a heuristic description of preloading strategies (Bismut & Straub, 2021). The optimization thereby yields optimized heuristic parameter values. We also investigate the influence of the assumed cost model on the obtained preloading plans.

The paper is structured as follows: Section 2 introduces the investigated embankment preloading problem in further details. Section 3 presents the preloading model. Section 4 summarizes the proposed decision-theoretic framework and Section 5 introduces the key concept of heuristic strategies. Section 6 present the specifics of the geotechnical and cost models in the numerical investigations, followed by the presentation of the results in Section 7. We discuss possible extensions of the investigation and adaptations of the method in Section 8.

2. Example application

To illustrate the proposed framework, we take the specific example introduced by Spross & Larsson (2021). We consider a section of an embankment built for the construction of Swedisch National Road 73, from southern Stockholm towards Nynäshamm. A cross section of the soil is shown in Figure 2.

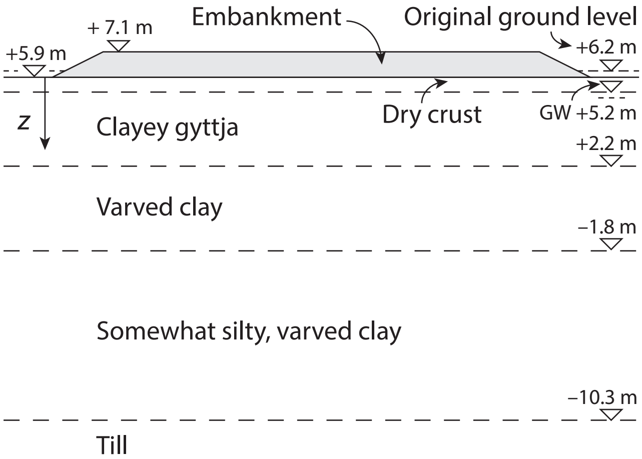


Figure 2: Cross-section of the soil under the planned embankment (from Spross & Larsson (2021), CC-BY 4.0, http://creativecommons.org/licenses/by/4.0/)

The considered engineering problem is the planning of the amount of surcharge loading the embankment during an available preloading time, t_{max}, within which an acceptable soil consolidation is to be reached. The engineering questions are: 1) What surcharge height should be used? 2) When is a load increase warranted during the preloading time, and if so, how much more should be added?

3. Geotechnical model and design requirements

In this section, we present the probabilistic model adopted to describe the evolution of soil settlement and resulting overconsolidation ratio, first under constant load then under multi stage loading. We extend the model described in Spross & Larsson (2021) to consider staged preloading, for the case of a surcharge increase. This geotechnical model considers 1) how primary compression settlement develops with time, due to the weight of the embankment and the surcharge, and 2) the effect of the unloading of the surcharge on the overconsolidation ratio (OCR). The parameters of the model are uncertain and are modeled as random variables. More detailed and complex models of settlement and consolidation behavior for staged construction are available in the literature (see, e.g., Walker & Indraratna, 2009; Yin et al., 2022), but we have opted for model simplicity to facilitate a focus on the optimization problem.

3.1. Settlement evolution

3.1.1. Constant surcharge

Under a constant load $\Delta\sigma$ and known soil properties, a settlement trajectory with time follows

$$S(t) = U(t)S_\infty, \tag{1}$$

where

$$U(t) = 1 - [1 - U_v(t)][1 - U_h(t)] \tag{2}$$

is the spatially averaged degree of consolidation at time t, and S_∞ is the predicted long-term primary compression settlement under load $\Delta\sigma$. The vertical consolidation component $U_v(t)$ is obtained from Terzaghi's consolidation theory. For the horizontal consolidation component $U_h(t)$ we apply Hansbo's well-established analytical PVD model (Hansbo, 1979), which considers the horizontal coefficient of consolidation, the PVD influence zone radius, as well as the effects of drain spacing, soil disturbance and well resistance. Due to the specific consolidation behaviour of soft clays, S_∞ is predicted as (Larsson & Sällfors, 1986)

$$S_\infty(\Delta\sigma) = \sum_{i=1}^{l} h_{cl,i} \Delta\varepsilon_i(\Delta\sigma), \tag{3}$$

where $h_{cl,i}$ is the thickness of the i-th clay layer. $\Delta\varepsilon_i$ is the strain increase caused by the load $\Delta\sigma$, which depends on parameters evaluated from constant-rate-of-strain tests, including the preconsolidation pressure and soil moduli (Spross & Larsson, 2021).

In the performed analyses, the embankment and surcharge are assumed to be of the same material, hence the load $\Delta\sigma$ is proportional to the material unit weight and to its total height.

3.1.2. Staged preloading

If the surcharge is increased by $\Delta\sigma_{add}$ after some preloading time, t_{add}, the adjusted settlement trajectory is modelled as:

$$S(t) = \begin{cases} U(t)S_\infty(\Delta\sigma), & \text{for } 0 \le t < t_{add} \\ U(t-t_{shift})S_\infty(\Delta\sigma+\Delta\sigma_{add}), & \text{for } t \ge t_{add}. \end{cases} \quad (4)$$

The first part of the trajectory is equivalent to Equation (1). The second part contains, due to the load increase, a recalculated, larger long-term primary consolidation settlement $S_{2,\infty} = S_\infty(\Delta\sigma + \Delta\sigma_{add})$ following Equation (3) and a corresponding degree of consolidation $U(t-t_{shift})$, for which a hypothetical zero degree of consolidation occurs at time $t_{shift} = t_{add} - t_0$. To determine t_0, we note that the settlement curve is continuous at t_{add}, which results in the degree of consolidation:

$$U(t_0) = \frac{U(t_{add})S_{1,\infty}}{S_{2,\infty}}, \quad (5)$$

where $U(t)$ is obtained from Equation (2). Figure 3 illustrates t_{shift} and the resulting settlement curve for staged preloading.

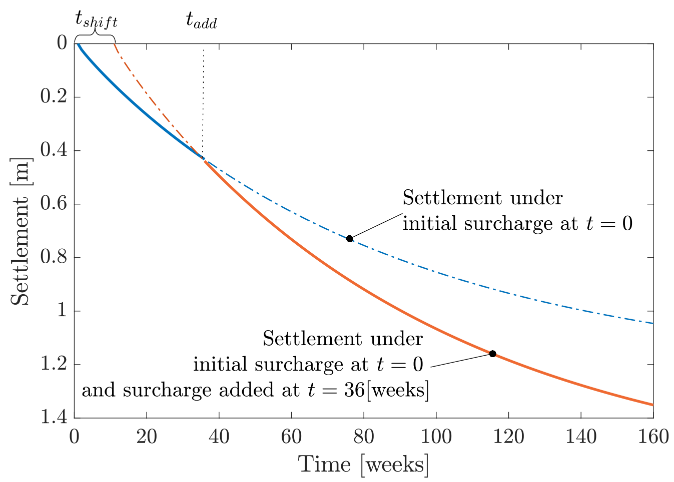


Figure 3: Effect of the added surcharge at time t_{add} on the settlement, where $t_{shift} = t_{add} - t_0$.

3.2. Overconsolidation ratio

3.2.1. Constant surcharge

The effect of secondary consolidation is considered through the OCR. The secondary consolidation is limited if the preloading achieves sufficient OCR in the middle of the clay stratum through unloading at time t (Alonso et al., 2000; Han, 2015). This quantity can be obtained as

$$OCR(t) = \frac{\sigma_0' + U(t)\Delta\sigma_{sur}}{\sigma_0' + U(t)\Delta\sigma_{emb}} \tag{6}$$

where σ_0' is the initial vertical stress increase in the middle of the clay stratum, $\Delta\sigma_{sur}$ is the vertical stress caused by the preloaded embankment (i.e. including the surcharge), and $\Delta\sigma_{emb}$ is the remaining stress increase directly after the unloading of the surcharge (see Figure 1).

3.2.2. Staged preloading

The effect of the added load on the OCR at unloading depends on the preloading time of both the initial and any added load. To our knowledge, there are no validated analytical models for this issue. Therefore, we use the following reformulation of Equation (6) to capture the effect on the OCR at the unloading at time t, when it occurs after a previous load increase at time t_{add}:

$$OCR(t) = \frac{\sigma_0' + U(t)\Delta\sigma_{sur} + \Delta U(t)\Delta\sigma_{add}}{\sigma_0' + U(t)\Delta\sigma_{emb}}, \tag{7}$$

where $\Delta U(t) = U(t - t_{shift}) - U(t_{add} - t_{shift})$. Consequently, the effect on the OCR of the added load will depend on the degree of consolidation achieved along the recalculated settlement trajectory after the load has been added. The OCR for staged preloading is depicted in Figure 4.

3.3. Uncertainties in the soil parameters

The presented soil consolidation model of Equations (1) to (7) depends on parameters for the soil properties and PVD design. The soil properties are modeled as random variables with an associated probability distribution either evaluated from constant-rate-of-strain (CRS) oedometer tests, or assigned based on engineering judgment when data on variability were not available. The parameters in Hansbo's PVD model (Hansbo, 1979) are assumed constant. The complete probabilistic model is described in detail by Spross & Larsson (2021) and is therefore omitted

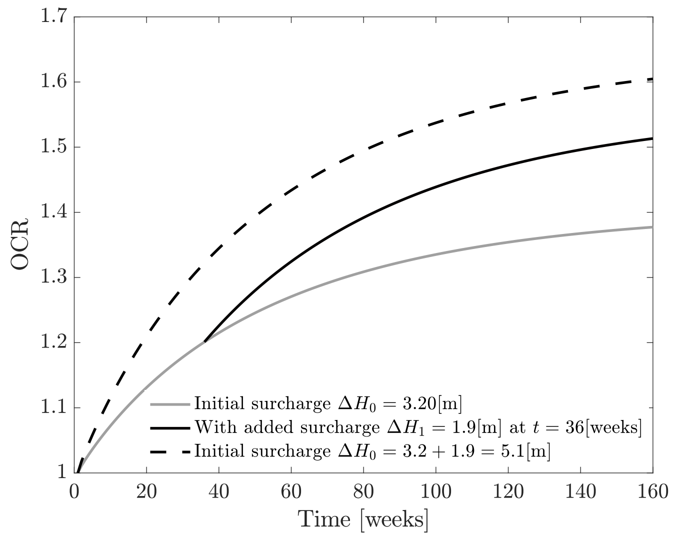


Figure 4: Effect of the surcharge added at time t_{add} on the OCR, using Equation (6) with initial surcharge $\Delta\sigma_{sur}$ corresponding to height ΔH_0 for the first part of the curve until $t = 36$[weeks] and Equation (7) with $\Delta\sigma_{add}$ corresponding to additional surcharge height ΔH_1. The resulting curve is located below the one for the case where the total surcharge (initial and additional) is applied directly at $t = 0$, with Equation (6).

for brevity, as the probabilistic soil characterization per se is not studied here. Random trajectory settlements are depicted in Figure 5.

3.4. Settlement and OCR requirements

The risk-based planning framework for optimal preloading described in Section 4 requires the definition of performance criteria, such that a preloading decision can be assessed in terms of its success to reach the desired goals. These goals are here expressed in terms of sufficient soil consolidation, through a settlement target s_{target}, and an OCR threshold, OCR_{target}.

3.4.1. Settlement target

Due to the uncertainty associated with the ground properties, the long term settlement S_∞ caused by the load of the completed embankment, $\Delta\sigma_{emb}$, is also uncertain. To ensure an acceptable residual (post-construction) primary consolidation settlement, Spross & Larsson (2021)

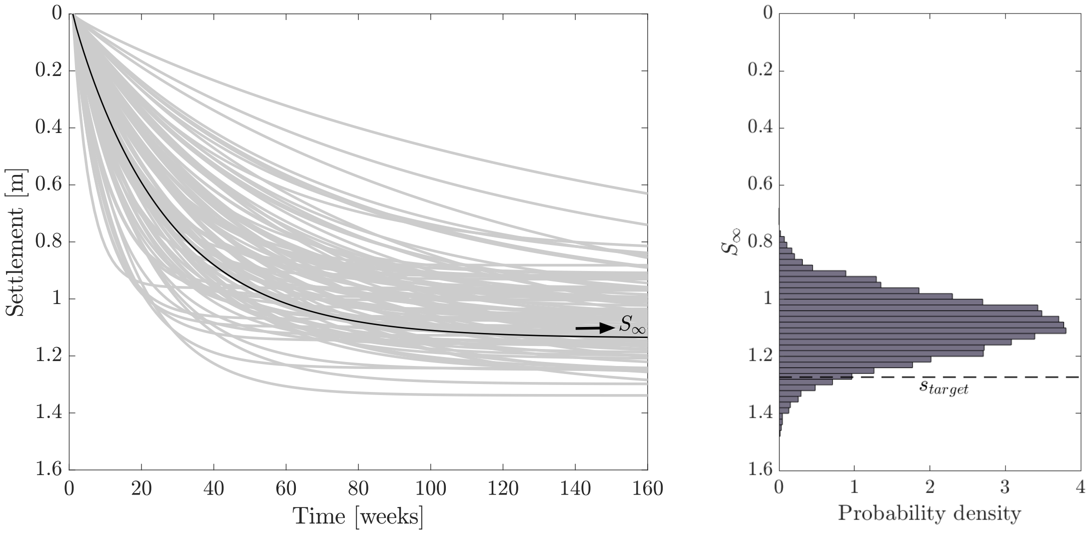


Figure 5: 100 sample soil settlement trajectories for an initial surcharge $h_0 = 0$[m] (no additional surcharge). One such trajectory is highlighted in black. For each trajectory, the value of the long-term settlement S_∞ is obtained with Equation (3). The histogram on the right shows the resulting distribution of S_∞. The condition $\Pr(S_\infty > s_{target}) = p_{FT} = 5\%$ (see Section 3.4.1) results in $s_{target} = 1.27$[m].

proposed that a target settlement, s_{target}, be attained during the preloading, such that

$$\Pr(S_\infty(\Delta\sigma_{emb}) > s_{target}) = p_{FT}, \tag{8}$$

where p_{FT} is an acceptable, fixed, probability. In the numerical investigations, it is set to 5% to represent a serviceability limit state.

By generating sample values of $S_\infty(\Delta\sigma_{emb})$ from the defined probabilistic model and Equation (3), s_{target} is obtained as the quantile value corresponding to p_{FT} (Figure 5). The value of s_{target} is thereafter used in the decision framework described in Section 4 to define penalty mechanisms.

3.4.2. OCR threshold

Residual secondary consolidation settlement (creep) can be significantly limited by ensuring that the OCR after unloading is sufficiently high (Alonso et al., 2000; Han, 2015). Here it is required that OCR exceeds $OCR_{target} = 1.10$ in the middle of the soft soil stratum after unloading of the surcharge. This is in line with the general technical requirements and guidance for geotechnical works issued by the Swedish Transport Administration (2013a,b).

4. Optimal preloading strategies

To find the optimal preloading strategy, we rely on the decision analysis framework of Raiffa & Schlaifer (1961), which formalized decision problems under uncertainty with varying information. This enables the optimization of the surcharge decisions, which can be done in a sequential manner based on measurements of the settlement. Further general information on sequential decision making can be found in Kochenderfer (2015).

4.1. Elements of the decision analysis

A decision analysis under uncertainty is based on a probabilistic model of the system, a model of the decision alternatives as well as a utility or cost function. These models are summarized in the following.

4.1.1. Probabilistic model

A complete probabilistic model describing the evolution of the system is required. It must account for the effects of actions affecting the system (see Section 4.1.2 below). This model must also reflect the uncertainty in information collection, through a likelihood function (Bismut & Straub, 2022).

In the investigated engineering problem, we use the soil consolidation model described in Section 3. Information on the state of the system is obtained as a measurement M_{t_1} of the settlement S_{t_1} at time t_1. The M_{t_1} is related to the true value of the settlement by an additive measurement error ε:

$$M_{t_1} = S_{t_1} + \varepsilon \tag{9}$$

Here, we restrict the numerical investigation to error-free measurement, i.e., $\varepsilon = 0$.

4.1.2. Decision alternatives

The decision alternatives, i.e., the available mitigating actions and other planning decisions, must be modeled. In the context of the preloading, these decision alternatives can include the height of the initial surcharge, as well as the timing and amount of additional surcharge that is applied later. The description of the available decision alternatives should also include operational constraints that must be accounted for in the planning process.

4.1.3. Utility and cost

The effects of a decision are evaluated in terms of utility, which reflects the preferences of the decision maker. Ultimately, the optimal decision is selected as the one that maximizes the expected utility. Assuming a risk-neutral context, the utility can simply translate to costs associated with the actions and the system performance. In this case, utility is expressed in monetary terms.

For the preloading example, we first quantify the cost $C_{sur,i}$ of adding a preloading surcharge of height ΔH_i. This cost should account for factors, such as material costs, mobilization costs, material availability at the time of the decision, and the need of berms for slope stability. The second cost component penalizes the project delay C_{delay}, which expresses the fact that sufficient settlement (s_{target}) has not been reached within a dedicated time period. In the example, this penalty is expressed as a function of the additional time required for reaching s_{target} (without further preloading intervention). Finally, the third cost component C_{OCR} quantifies the consequences of residual secondary consolidation settlement (creep) caused by insufficient overconsolidation at time of unloading (see Section 3.4.2). Thus, the C_{OCR} reflects a reduced serviceability of the superstructure.

The total cost C_{tot} incurred at the completion of the preloading operation is the sum of the three cost components:

$$C_{tot} = \sum_i C_{sur,i} + C_{delay} + C_{OCR} \tag{10}$$

If relevant, discounting can be used to reflect the decreasing value of an investment over time, but this effect is however ignored here.

4.2. Decision settings and influence diagrams

With the above elements specified, a decision setting (DS) is defined. A typical compact graphical representation of a DS is the influence diagram (ID) (Jensen et al., 2007), which depicts the available decisions/actions and the utilities (costs). Round nodes represent uncertain outcomes (which are described by the probabilistic model), square nodes are the decisions and lozenge-shaped nodes are the utility. The nodes are connected by directed edges, which represent stochastic, causal and monetary dependence. When the problem involves sequential decisions, future information cannot influence past decisions.

The decision setting is usually determined by operational constraints, as well as the level of complexity of the considered decision sequence. For this study we construct IDs for three different decision settings.

DS #1: Surcharge applied at $t = 0$

In DS #1, we consider the case where the surcharge is applied only at the time of constructing the embankment, i.e., at $t = 0$. The only decision variable is the height ΔH_0 of this surcharge. The settlement at time t, S_t, and the achieved overconsolidation ratio if unloaded at time t, OCR_t, are both probabilistic quantities, which depend on the applied surcharge as per the models of Section 3.

The overall decision process is summarized by the ID of Figure 6. The square node ΔH_0 indicates that first a value of ΔH_0 is chosen, at a cost $C_{sur,0}(\Delta H_0)$. The now fixed ΔH_0 influences the evolution of the settlement S_t and the overconsolidation ratio at unloading OCR_{fin} as well as the time t_{target} when the target settlement is reached, defined as $S(t_{target}) = s_{target}$. Monetary consequences due to project delay and residual creep result from these quantities. The cost model used to quantify the consequences is presented in Section 6.2.

DS #2 and DS #3: Surcharge applied at $t = 0$ *and adjusted at time* t_1

DS #2 and DS #3 consider that there is an opportunity to add a surcharge of height ΔH_1 at a fixed time t_1, on top of the initial surcharge height ΔH_0. The decision on how much to add is based on a measurement M_{t_1} of the settlement at time t_1 (see Section 4.1.1). The overall decision

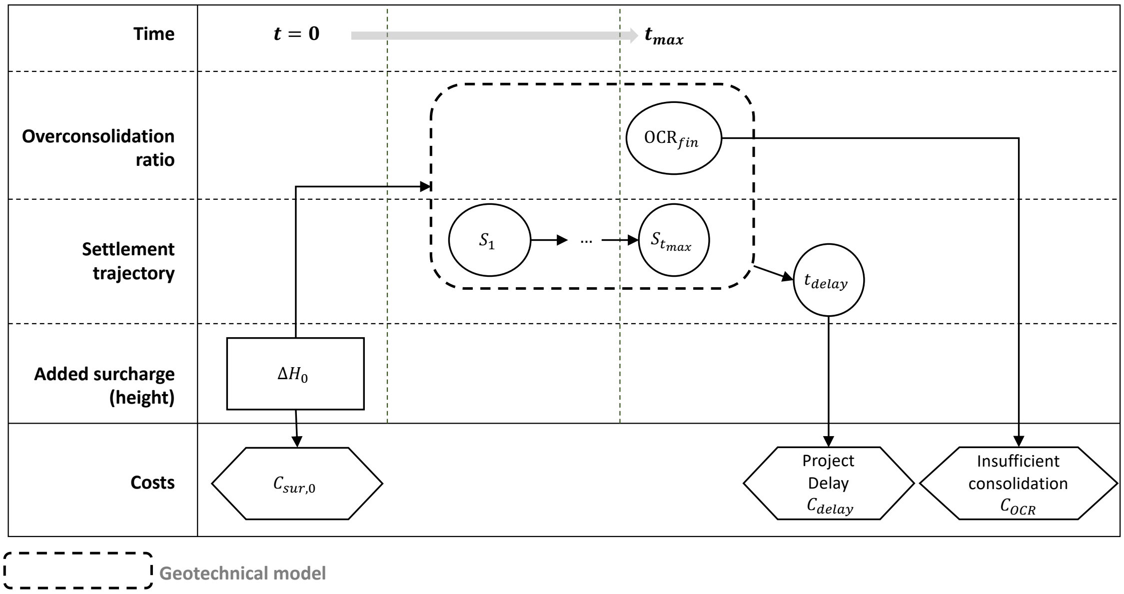


Figure 6: Influence diagram for DS #1. Optimization of the initial surcharge. The interaction between the decision on the initial surcharge height ΔH_0 and the geotechnical model is represented in a simplified manner.

process is summarized by the ID depicted in Figure 7. In DS #2, the time t_1 is fixed and cannot be influenced by the decision maker, whereas in DS #3, this time can be chosen and optimized.

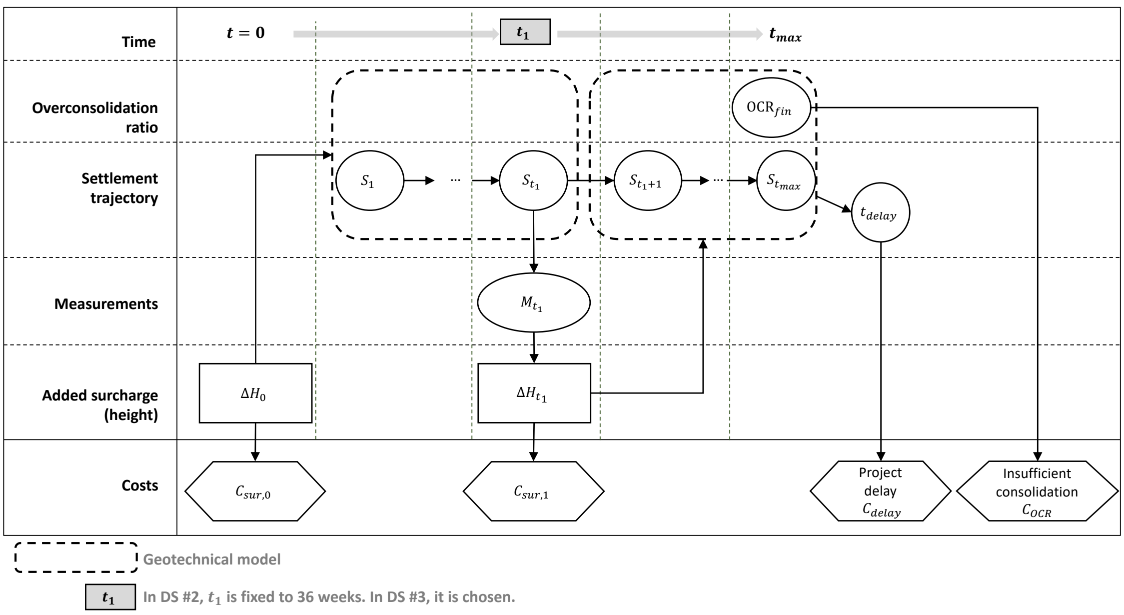


Figure 7: Influence diagram for DS #2 and DS #3. The interactions between the decisions on the initial and added surcharge heights, ΔH_0 ΔH_1, and the geotechnical model are represented in a simplified manner.

4.3. Optimal decision making

The most desirable outcome of the decision process is the one with the lowest cost. Due to the uncertain nature of the soil parameters, the outcomes of a sequence of decisions are uncertain, hence so is the total cost. The optimal sequence of decisions is therefore that which result in the minimum expected total cost (Raiffa & Schlaifer, 1961). For DS #1, the optimal decision for ΔH_0 is therefore defined as:

$$\Delta H_0^* = \arg\min \mathbf{E}\left[C_{tot}(\Delta H_0)\right], \tag{11}$$

where $\mathbf{E}\left[C_{tot}(\Delta H_0)\right]$ is the expected value of the total cost evaluated with Equation (10), when an initial preloading surcharge of height ΔH_0 is applied. This expected total cost thus accounts for the associated risk $\mathbf{E}[C_{delay}(\Delta H_0)] + \mathbf{E}[C_{OCR}(\Delta H_0)]$ of not achieving the desired settlement or overconsolidation ratio within the available preloading time.

The formulation of the optimization problem is not as straightforward for DSs, where there are one or more opportunities to adjust the surcharge after the initial surcharge is applied, i.e. DS #2 and DS #3. In these sequential decision problems, the optimal actions depend on the past observations. Therefore, one must find the optimal function that maps past observations to actions. In general, this type of problem is hard to solve and an exact solution becomes intractable with increasing number of decision or observation steps (Papadimitriou & Tsitsiklis, 1987). Approximate solutions are possible, e.g., via partially observable Markovian decision processes (POMDP) or reinforcement learning (Porta et al., 2005; Roy et al., 2005; Silver & Veness, 2010; Mnih et al., 2013; Memarzadeh & Pozzi, 2016; Papakonstantinou et al., 2018; Andriotis & Papakonstantinou, 2019).

To solve the general sequential decision problem, it is convenient to define *preloading strategies* $\mathcal{S}$, which compactly prescribe the sequence of decisions. A strategy consists of a set of rules which prescribes how much surcharge to add at any time as allowed by the DS. For example, for DS #1, a strategy simply prescribes the surcharge height at time $t = 0$; for DS #2, it prescribes the surcharge height at time $t = 0$ and gives a rule at time t_1, which can be based on settlement measurements, to adjust the surcharge. In DS #3, the strategy additionally prescribes the time $t = t_i$ at which to collect the settlement measurement and adjust the surcharge.

Generalizing the notation to any preloading strategy $\mathcal{S}$, the expected total cost associated with a preloading strategy $\mathcal{S}$ is thus evaluated as:

$$\mathbf{E}\left[C_{tot}(\mathcal{S})\right] = \mathbf{E}[C_{sur}(\mathcal{S})] + \mathbf{E}[C_{delay}(\mathcal{S})] + \mathbf{E}[C_{OCR}(\mathcal{S})]. \tag{12}$$

The optimal preloading problem exposed in Section 4.3 is equivalent to finding the preloading strategy that minimizes the expected total cost:

$$\mathcal{S}^* = \arg\min_{\mathcal{S}} \mathbf{E}\left[C_{tot}(\mathcal{S})\right] \tag{13}$$

In general, $\mathbf{E}\left[C_{tot}(\mathcal{S})\right]$ cannot be evaluated analytically. A Monte Carlo (MC) approximation can instead be obtained using the geotechnical model of Section 3. The latter enables the generation of n_{MC} random settlement trajectories, $\boldsymbol{S}_{\boldsymbol{t}}^{(k)}$, and OCR at unloading $OCR_{fin}^{(k)}$, obtained from surcharge sequences $\Delta H_0^{(k)}$, $\Delta H_1^{(k)}$, etc., with $1 \leq k \leq n_{MC}$. A total cost can be computed for each of these trajectories as per Equations (10), (16), (18) and (19). The MC approximation of the expected total cost of a preloading strategy $\mathcal{S}$ is therefore

$$\mathbf{E}\left[C_{tot}(\mathcal{S})\right] \simeq \frac{1}{n_{MC}} \sum_{k=1}^{n_{MC}} C_{tot}\left(\boldsymbol{S}_{\boldsymbol{t}}^{(k)}, OCR_{fin}^{(k)}\right) \tag{14}$$

The estimate improves with the number of samples n_{MC}.

5. Heuristics for optimal preloading strategies

The problem of finding the best strategy is equivalent to finding the best sequence of decision and an exact solution to Equation (13) is still intractable in general. To address this challenge, we reduce the space of possible strategies that are considered in the optimization, following Bismut & Straub (2022). Since strategies are sets of rules, the proposed approach considers only strategies, which are described by specific sets of rules. We call these *heuristics*. The heuristic chosen for this approach is typically formulated with simple statements (the rules), in which a number of parameters $\mathbf{w} = [w_1; w_2; ...; w_n]$ intervene. For example, we define the following heuristic for DS #2:

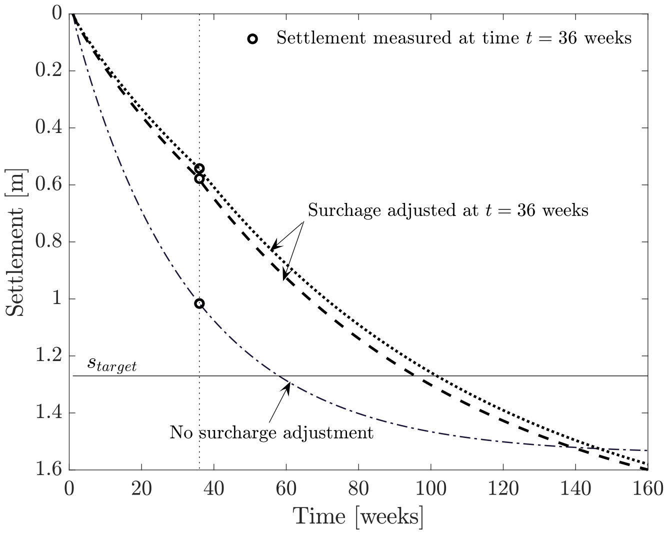


Figure 8: Three sample trajectories for a strategy parametrized with Heuristic 2A (Section 6.3.2), with $h_0 = 0.95$m, $h_1 = 1.04$m and $s_{th} = 0.77$m. The time at which the curves intersect with the level s_{target} corresponds to t_{target}. For $t_{max} = 72$[weeks], we see that only one of these trajectories satisfies $t_{target} < t_{max}$ and does not lead to project delay.

- The initial surcharge ΔH_0 is h_0;

- The additional surcharge ΔH_1 at time $t_1 = 36$ weeks is h_1 if the measured settlement at this time is lower than a threshold s_{th}.

The parameters $\mathbf{w}$ for this heuristic are h_0, h_1 and s_{th}. In this DS, t_1 is fixed to 36 weeks. A preloading strategy following this heuristic, assigned chosen parameters $h_0 = 0.94$m, $h_1 = 1.04$m and $s_{th} = 0.77$m, will react to different trajectories as shown in Figure 8. The total cost incurred will depend on a) the strategy and b) the settlement occurring.

The expected costof a strategy with fixed parameters can be estimated with Equation (14). For a given heuristic, there is a set of parameter values that optimize the expected cost. We call the associated strategy the *optimal heuristic strategy*. Thus, for a given heuristic and associated parameters $\mathbf{w} = [w_1; w_2; ...; w_n]$, the preloading problem is reduced to finding

$$\mathbf{w}^* = \arg\min \mathbf{E}\left[C_{tot}(\mathcal{S}(\mathbf{w}))\right] \tag{15}$$

As the heuristic formulation of the optimization problem operates in a restricted strategy space, it yields a sub-optimal preloading strategy. However, the heuristic parametrization enables the inclusion of operational constraints and provides easily interpretable strategies. Furthermore, the definition of preloading strategies with heuristics makes sense from the point of view of geotechnical engineering practice, as most preloading strategies would indeed be defined with such simple rules. In addition, several heuristics can be compared and the better-performing strategy selected. In the numerical investigations we discuss the impact of different heuristic choices, in particular the impact of increasing the number of heuristic parameters.

The optimal parameter values $\mathbf{w}^*$ are the solution of a noisy optimization problem where the objective function is expressed as an expected value (Rubinstein & Kroese, 2004), for which no analytical expression exists. The crudest approach to this problem is to search among preselected values of heuristic parameters (for instance on a grid), estimate the expected cost with Equation (14) at each point with a sufficiently high number of samples n_{MC} and finally select the most cost-efficient strategy. The disadvantages of this approach are that the search is restricted to a finite number of strategies; equal computational budget, n_{MC}, is attributed to all parameter values, including those that are sub-optimal and yield a high expected cost; and for a high number n of heuristic parameters the grid search leads to an infeasible compuational effort.

A more efficient approach is a sampling-based optimization, such as presented in Appendix A, which was previously developed for this purpose in Bismut & Straub (2021) and based on the cross-entropy (CE) method (Rubinstein & Kroese, 2004). An initial sampling density over the heuristic parameters $\mathbf{w}$ is chosen, for instance a multivariate Gaussian distribution. At each iteration and until convergence is reached (see Figure 9) – or until a maximum number of iterations is exceeded – n_S sample sets of parameter values are generated from the sampling density. For each sample set, the expected cost of the associated strategy is evaluated with n_{MC} samples. The sample sets are ranked in increasing order of expected cost. The parameters of the CE sampling density for the next iteration are fitted to the top n_{CE} sample sets, the elite samples. We have previously demonstrated this method on other sequential decision planning problems (Bismut & Straub, 2021; Bismut et al., 2022). The method stands out for the simplicity of its implementation and robustness.

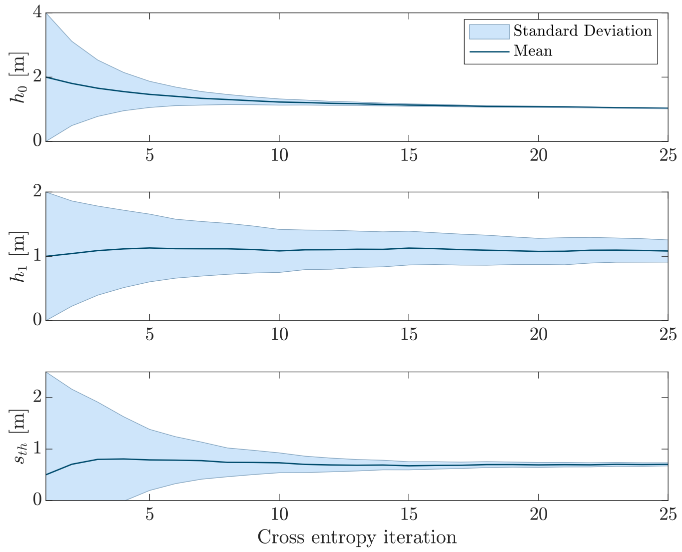


Figure 9: Convergence of heuristic parameters in the CE optimization for Heuristic 2A defined in Section 6.3.2.

6. Numerical investigations

6.1. Probabilistic model setup

The probabilistic model setup is described in Spross & Larsson (2021). The settlement target is computed for $p_{FT} = 0.05$, and is obtained as $s_{target} = 1.27[m]$.

6.2. Cost model

The $C_{sur,i}$ corresponds to the cost of adding surcharge of height ΔH_i. It increases with the total surcharge height, and accounts for the cost of berms needed to ensure slope stability (see Figure 1). It is evaluated from the cost of total surcharge height H_{tot}:

$$C_{sur}(H_{tot}) = \begin{cases} H_{tot} \cdot c_{sur} & \text{if } H_{tot} \leq 1m \\ 1.25 \cdot H_{tot} \cdot c_{sur} & \text{otherwise.} \end{cases} \quad (16)$$

The cost attributed to each increase ΔH_i of surcharge on top of existing surcharge H_{tot} is computed as

$$C_{sur,i}(\Delta H_i) = (C_{sur}(H_{tot} + \Delta H_i) - C_{sur}(H_{tot})) \cdot f_{add,i} \quad (17)$$

where the factor $f_{add,i} \geq 1$ accounts for additional costs incurred by increasing the surcharge at a later time $t > 0$. Note that the cost of the remaining embankment material is not included here, as it is the same for all possible scenarios.

In the model, project delay occurs when the settlement trajectory either does not meet s_{target} within the available preloading time t_{max} ($t_{target} > t_{max}$) or is unable to meet s_{target} at all ($t_{target} > t_{lim}$) (see Figure 8). The associated penalty is

$$C_{delay}(t_{target}) = \begin{cases} 0 & \text{if } t_{target} \leq t_{max} \\ c_{delay} \cdot (\min(t_{lim}, t_{target}) - t_{max}) & \text{otherwise,} \end{cases} \tag{18}$$

where c_{delay} represents the penalty per week of delay.

Finally, the penalty associated with residual creep settlement in the completed structure due to insufficient OCR (see Section 3.4.2) is evaluated with the logistic function

$$C_{OCR}(OCR_{fin}) = \frac{c_{OCR}}{1 + \exp\left(-\frac{1.075 - OCR_{fin}}{4.5 \cdot 10^{-3}}\right)} \tag{19}$$

where OCR_{fin} is the OCR at unloading at time t_{target} or t_{lim} if the settlement target has not been achieved in time. This smoothed step function approaches c_{OCR} when $OCR_{fin} < 1.05$, and 0 when $OCR_{fin} > 1.1$.

The cost factors c_{sur}, c_{delay} and c_{OCR} for the initial numerical investigation are given in Table 1. The effect of varying these factors is shown in Section 7.1.

6.3. Heuristic parametrizations

We investigate the following heuristics for the different DSs. The heuristic parameters for each defined heuristic are indicated in bold.

6.3.1. DS #1

As explained in Section 4.2, the optimization for this setting only consists in optimizing the initial surcharge height ΔH_0, thus the corresponding heuristic, with single heuristic parameter h_0, is simply

Table 1: Parameters of the cost model

Cost factor	Value
c_{sur}	$3.45 \cdot 10^6 [SEK/m]$
c_{delay}	$3 \cdot 10^5 [SEK/week]$
c_{OCR}	$2 \cdot 10^7 [SEK]$
$f_{add,0}$	1
$f_{add,1}$	1

Heuristic 1: $\mathbf{h_0} \geq 0$

1. $\Delta H_0 = \mathbf{h_0}$.

6.3.2. DS #2

For DS #2, we investigate the performance of two different heuristics in approximating the optimal preloading strategy. A preloading strategy described with Heuristic 2A specifies the initial surcharge height, and adjusts it by adding a surcharge height if the measured settlement is lower than a threshold.

Heuristic 2A: $\mathbf{h_0} \geq 0$, $\mathbf{h_1} \geq 0$, $\mathbf{s_{th}} \geq 0$

1. At time $t = 0$, add surcharge of height $\Delta H_0 = \mathbf{h_0}$.
2. Obtain measurement m_{t_1} at time $t_1 = 36$[weeks].
3. If $m_{t_1} < \mathbf{s_{th}}$, add surcharge $\Delta H_1 = \mathbf{h_1}$. Otherwise $\Delta H_1 = 0$.

With Heuristic 2B, the strategy adjusts the height of the added surcharge based on the difference d between the measured settlement and the threshold. This height adjustment is defined by a sigmoid function varying between 0 and maximum added height h_1, characterized by a curve steepness a. When $a = 0$, this sigmoid function is a step function.

Heuristic 2B: $\mathbf{h_0} \geq 0$, $\mathbf{h_1} \geq 0$, $\mathbf{s_{th}} \geq 0$, $\mathbf{a} \leq 0$

1. At time $t = 0$, add surcharge of height $\Delta H_0 = \mathbf{h_0}$.
2. Obtain measurement m_{t_1} at time $t_1 = 36 weeks$.
3. Compute $d = m_{t_1} - \mathbf{s_{th}}$
4. Add surcharge $\Delta H_1 = \begin{cases} 0 & d \leq \mathbf{a} \\ 2\mathbf{h_1}\left(\frac{d-\mathbf{a}}{2\mathbf{a}}\right)^2 & \mathbf{a} \leq d \leq 0 \\ \left(1 - 2\left(\frac{d-\mathbf{a}}{2\mathbf{a}}\right)^2\right)\mathbf{h_1} & 0 \leq d \leq -\mathbf{a} \\ \mathbf{h_1} & d \geq -\mathbf{a} \end{cases}$.

6.3.3. DS #3

Heuristic 3 is the same as 2B, with the additional freedom to choose the time t_1 at which the settlement is measured and the surcharge height is adjusted. The t_1 is thus an additional heuristic parameter.

Heuristic 3: $\mathbf{h_0} \geq 0$, $\mathbf{h_1} \geq 0$, $\mathbf{s_{th}} \geq 0$, $\mathbf{a} \leq 0$, $\mathbf{t_1} \in \{1, 2, 3, ..., t_{max}\}$

1. At time $t = 0$, add surcharge of height $\Delta H_0 = \mathbf{h_0}$
2. Obtain measurement m_{t_1} at time $\mathbf{t_1}$.
3. Compute $d = m_{t_1} - \mathbf{s_{th}}$
4. Add surcharge $\Delta H_1 = \begin{cases} 0, & d \leq \mathbf{a} \\ 2\mathbf{h_1}\left(\frac{d-\mathbf{a}}{2\mathbf{a}}\right)^2, & \mathbf{a} \leq d \leq 0 \\ \left(1 - 2\left(\frac{d-\mathbf{a}}{2\mathbf{a}}\right)^2\right)\mathbf{h_1}, & 0 \leq d \leq -\mathbf{a} \\ \mathbf{h_1}, & d \geq -\mathbf{a}. \end{cases}$.

6.4. Computational setup

For the CE method, we fix $n_{CE} = 100$, $n_E = 30$ and $n_{MC} = 10$. On a 8-core CPU 3.2GHz machine, optimizing the heuristic parameters for a given heuristic takes ca. 4min. The expected cost of the resulting optimized strategy is evaluated with $n_{MC} = 10^4$ samples.

7. Results

We apply the CE method to obtain the optimal parameter values and associated expected costs for the different DSs and heuristics defined above, assuming the cost model of Table 1. The results are summarized in Table 2.

Table 2: Optimal heuristic parameters and associated expected costs

		DS #1	DS #2		DS #3
Parameter	Unit	Heuristic 1	Heuristic 2A	Heuristic 2B	Heuristic 3
h_0	[m]	1.05	0.98	0.96	0.95
h_1	[m]	-	1.06	1.08	1.81
s_{th}	[m]	-	0.71	0.73	0.37
a	[m]	-	-	-0.15	-0.28
t_1	[weeks]	-	$36^{(*)}$	$36^{(*)}$	20
Expected cost	$[10^6 SEK]$	8.11	6.54	6.29	6.06
Std. dev. cost	$[10^6 SEK]$	7.4	6.3	6.0	5.6

$^{(*)}$Value is not optimized but fixed

The expected costs of the optimal heuristic strategies obtained for each of the DS decrease from DS #1 to DS #3. This is in agreement with the fact that DS #1 is more restrictive in terms of available actions than DS #2, and in turn DS #2 is more restrictive (because the adjustment time is fixed) than DS #3. Table 2 also reports the estimated standard deviation of the total cost. For the investigated heuristics, the coefficient of variation of the total cost for the optimal strategy varies around 95%. The standard error of the MC estimates of the expected costs is therefore 1%, which ensures a sufficient accuracy to rank the heuristics according to the estimated expected cost of their optimal strategies.

The optimal initial surcharge prescribed by Heuristic 1 in DS #1 is higher than the initial surcharge prescribed in DS #2 and DS #3. This shows that the heuristics chosen for DS #2 and DS

#3 exploit the fact that measurement information enables an optimized adjustment of surcharge.

For DS #2, we note that Heuristic 2B performs better than Heuristic 2A in terms of expected cost; hence the smoothed step function for the selection of the adjusted load is a better heuristic than the simple step function.

Figure 10 depicts the breakdown of the costs for each optimal heuristic strategy. We observe that Heuristic 3 yields a lower risk of delay than Heuristic 2A and 2B and a lower expected total cost, even though it applies on average a higher total surcharge. Therefore, the choice of time t_1 to adjust the surcharge plays a significant role in efficiently controling the settlement. The expected penalty associated with insufficient OCR is here negligible in comparison with the other cost components, for all heuristics.

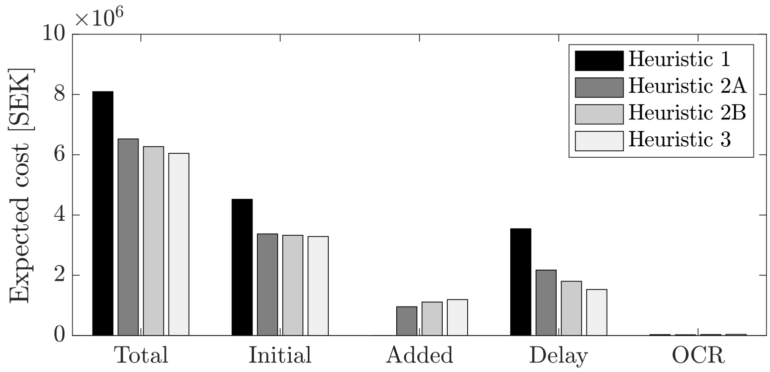


Figure 10: Breakdown of the expected cost of the optimal strategies for the different DSs and heuristic.

Figure 11 illustrates the effect of adjusting the surcharge at time $t_1 = 36$ on the settlement trajectory, following the optimal strategy for Heuristic 2A. The distribution of the settlement at time t_{max} is obtained from 10^4 sample trajectories for both the case where only the initial surcharge is applied and not adjusted at $t = 36$ weeks and the case where the surcharge is adjusted according to the optimal strategy. With the load adjustment action, the settlement trajectories that already reach the target at t_{max} with the sole initial load are unaffected, while a portion of trajectories which would not have achieved s_{target} at t_{max} are now compliant, i.e., the probability $\Pr(S_{t_{max}} < S_{target})$ decreases by enabling the adjustment of the surcharge. Most of the corrected trajectories will nevertheless incur a delay penalty, which is optimal under the assumed cost model of Table 1.

The effect of the different heuristics on the final settlement at time t_{max} and on the OCR at

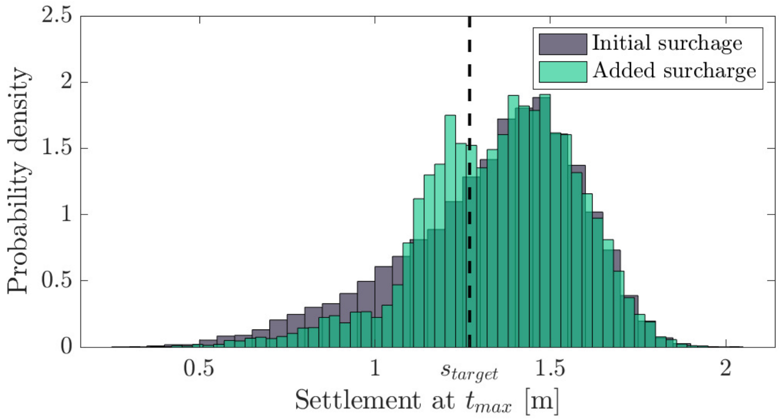
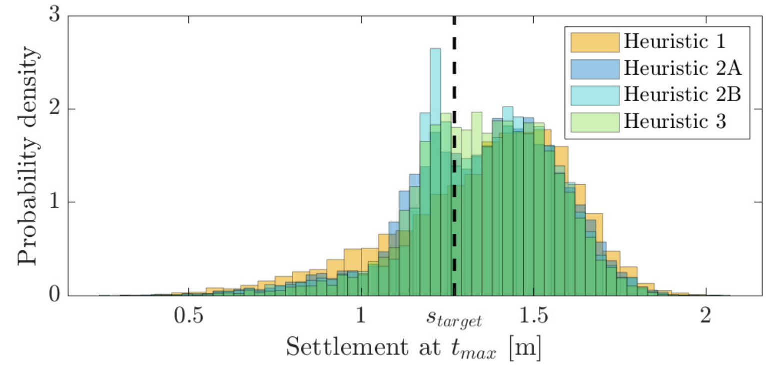


Figure 11: Distribution of settlement at t_{max} for the optimal strategy for DS #2, Heuristic 2A, obtained from 10^4 sample settlement trajectories. The first histogram represents the distribution of the settlement if only the initial surcharge of height $h_0 = 0.95$m is applied. The second histogram shows the distribution of the settlement obtained by adjusting the surcharge at $t = 36$ weeks, as prescribed by the strategy (see Table 2). s_{target} is also indicated.

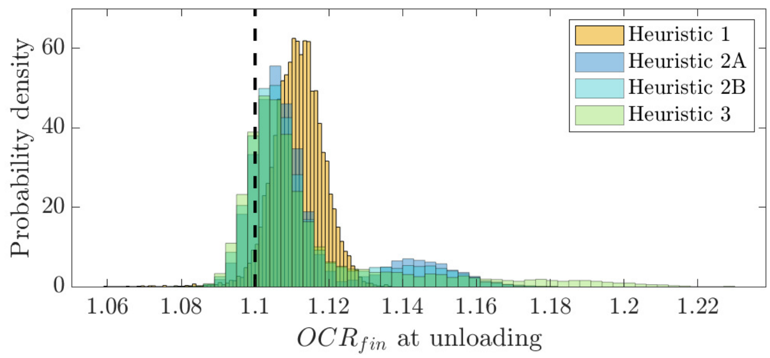
unloading is depicted in Figures 12a and 12b. Heuristics 2A, 2B and 3 can be distinguished from Heuristic 1, where the preloading is only added at $t = 0$. The uncertainty in the settlement reduces when the surcharge is adjusted based on the measured settlement, and the probability that $S_{t_{max}}$ is larger than s_{target} increases from Heuristic 1 to Heuristic 3. It is worth noting that the optimal strategies for Heuristics 2A, 2B and 3 result in a larger probability that the OCR at unloading is smaller than the critical value 1.1, compared to Heuristic 1, hence these heuristics can balance both penalties associated with insufficient settlement and OCR against the applied surcharge in a more efficient manner.

7.1. Sensitivity to the cost model

We vary the parameters c_{delay} and c_{OCR} and $f_{add,1}$ of the cost model (Table 1). Figure 13 compares the expected cost functions for DS #1 for the original cost model of Table 1 against the case where the delay penalty factor c_{delay} is doubled and the case where the consequences for insufficient OCR are increased 10-fold. Varying $f_{add,1}$ does not affect the expected costs within DS #1. We note that the location of the minimum expected cost is not as sensitive to an increased penalty for insufficient OCR as with an increased factor c_{delay}, which results in a higher initial surcharge.



(a)



(b)

Figure 12: Distribution of (a) settlement achieved at t_{max} and (b) of the OCR at unloading for the optimal heuristic strategies (see Table 2). The area of the histograms to the left of the dotted line represents for each optimal heuristic strategy, in (a) the probability $\Pr(S_{t_{max}} < s_{target})$, and in (b) the probability $\Pr(OCR_{fin} < 1.1)$.

Tables 3 to 5 report the optimal heuristic parameters and expected costs for the various cost models. The expected costs for the different heuristics, under different cost models still follow the cost ranking observed for the original cost model in Table 2.

Increasing the surcharge penalty $f_{add,1}$ in Table 3 notably results in a later optimal addition of the surcharge in Heuristic 3, in comparison to the optimal strategy for Heuristic using the original cost model, as shown in Table 2. For the increased OCR penalty in Table 5, the coefficient of variation of the total cost when applying the optimal heuristic strategies is significantly lower than for the original cost model, around 80%.

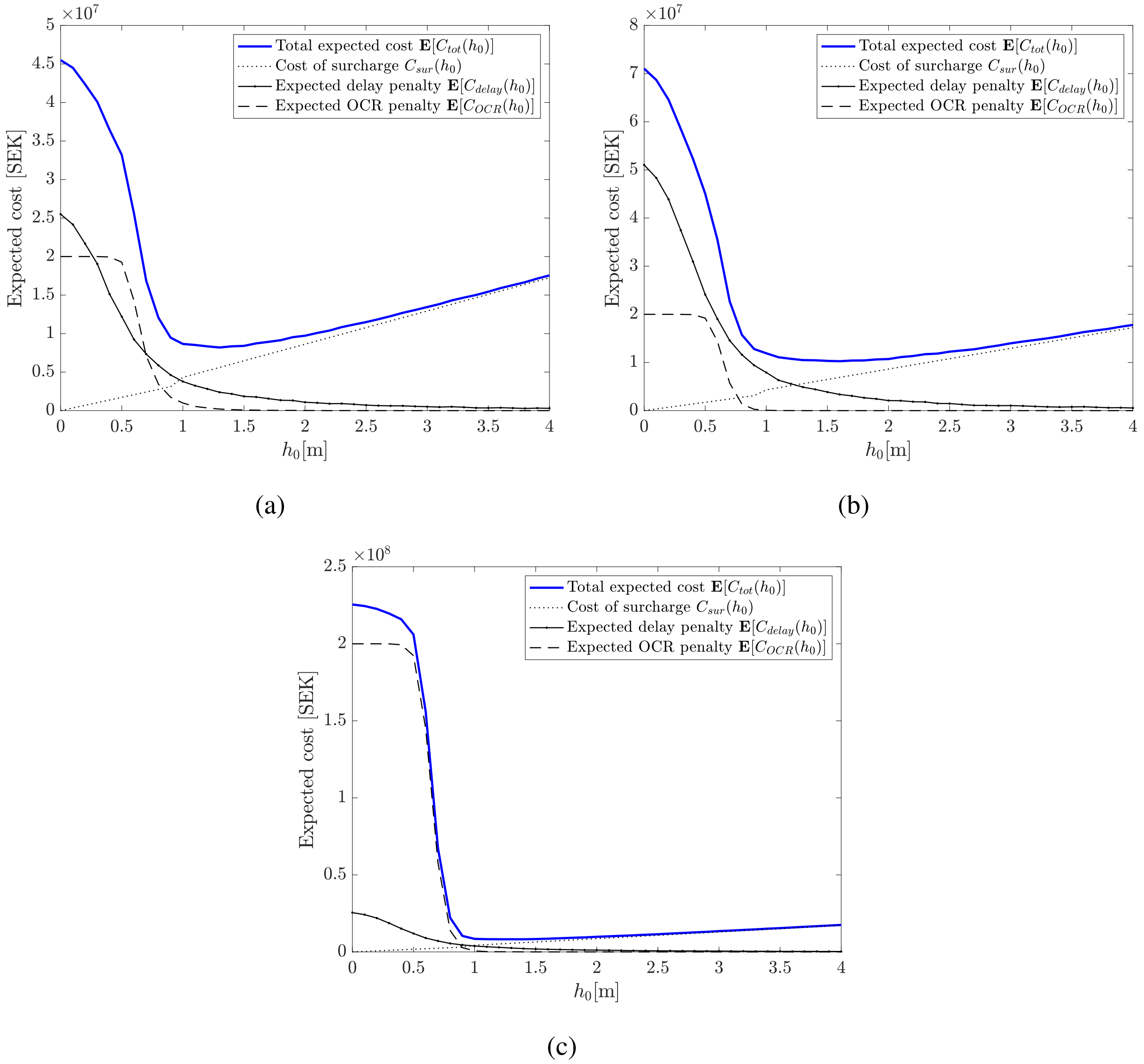


Figure 13: Expected costs for DS #1 as a function of $\Delta H_0 = h_0$: (a) for the original cost model ; (b) for an increased factor $c_{delay} = 6 \cdot 10^5$ SEK/week ; and (c) for an increased factor $c_{OCR} = 10^8$SEK.

8. Discussion

8.1. Designing the strategies

The preloading problem is re-formulated as a sequential decision problem, with different decision settings. Preloading strategies are described through heuristics with associated parameters. We observe that the more flexibility in decision the heuristic provides, the more cost efficient the

Table 3: Optimal heuristic parameters with an increased surcharge addition penalty factor $f_{add,1} = 1.3$

		DS #1	DS #2		DS #3
Parameter	Unit	Heuristic 1$^{(**)}$	Heuristic 2A	Heuristic 2B	Heuristic 3
h_0	[m]	1.05	0.97	0.91	0.95
h_1	[m]	-	1.02	1.19	1.86
s_{th}	[m]	-	0.69	0.64	0.61
a	[m]	-	-	-0.27	-0.40
t_1	[weeks]	-	$36^{(*)}$	$36^{(*)}$	40
Expected cost	[$10^6 SEK$]	8.11	6.97	6.92	6.84
Std. dev. cost	[$10^6 SEK$]	7.4	7.0	6.9	6.8

$^{(*)}$Value is not optimized but fixed

$^{(**)}$Values from Table 2

Table 4: Optimal heuristic parameters for increased $c_{delay} = 6 \cdot 10^5$[SEK/week]

		DS #1	DS #2		DS #3
Parameter	Unit	Heuristic 1	Heuristic 2A	Heuristic 2B	Heuristic 3
h_0	[m]	1.38	0.96	0.95	0.97
h_1	[m]	-	1.25	1.35	2.4
s_{th}	[m]	-	0.81	0.73	0.40
a	[m]	-	-	-0.23	-0.44
t_1	[weeks]	-	$36^{(*)}$	$36^{(*)}$	23
Expected cost	[$10^6 SEK$]	10.23	7.90	7.85	7.34
Std. dev. cost	[$10^6 SEK$]	11.28	10.0	10.1	9.0

$^{(*)}$Value is not optimized but fixed

Table 5: Optimal heuristic parameters for increased $c_{OCR} = 10^8$[SEK]

		DS #1	DS #2		DS #3
Parameter	Unit	Heuristic 1	Heuristic 2A	Heuristic 2B	Heuristic 3
h_0	[m]	1.14	1.09	1.07	0.99
h_1	[m]	-	0.85	2.48	2.63
s_{th}	[m]	-	0.71	0.55	0.53
a	[m]	-	-	-0.44	-0.51
t_1	[weeks]	-	$36^{(*)}$	$36^{(*)}$	39
Expected cost	[$10^6 SEK$]	8.17	7.26	7.04	6.67
Std. dev. cost	[$10^6 SEK$]	8.5	5.6	5.5	5.8

$^{(*)}$Value is not optimized but fixed resulting optimal heuristic strategy is.

Other heuristics than those proposed can be investigated, and might result in lower expected costs. For example, one might replace the sigmoid function of Heuristic 2B by another function. As settlement measurement is typically available at weekly intervals, a heuristic could be formulated such that the adjusted surcharge at time t_1 depends on an observed trend. In this case, how the measurements are processed for the purpose of decision-making, hence the trend prediction model, belongs to the definition of the heuristic. Ultimately, one could define a heuristic to address the problem where continuous settlement measurement is available, with near-real-time decision support.

The advantage of the heuristic approach to the planning of preloading decisions is that the resulting strategies are interpretable, since the decision rules are explicitly defined through the chosen heuristic. This also also entails that the heuristic can encode geotechnical expertise. The flexibility in the formulation of the decision setting through the influence diagrams and the cost functions also enables the analyst to integrate additional constraints. For instance, the uncertainty in the availability of preloading material could be explicitly modeled, such that there is a certain probability of obtaining the requested material at a given point in time. We note that the coefficient of variation of the total cost is large, around 100%. If the decision-maker wanted to prioritize strategies that reduced this variability, a risk-averseness behavior could be included in the objective function of Equation (13).

8.2. Integration with the observational method

The decision-theoretical framework described in this paper is suitable to apply in combination with the observational method, which was first defined as a design approach by Peck (1969) and today is accepted into design codes like Eurocode 7 (CEN EN 1997-1:2004). The observational method implies that the geotechnical engineer establishes a monitoring plan with thresholds that trigger prepared design changes specified in an action plan, thereby adjusting the initial design to fit better to the actual ground conditions.

In the context of a sequential decision problem, such thresholds and design changes can be formulated as heuristics, allowing the geotechnical engineer not only to compare conceptually different options of monitoring and action plans, but also to optimize their included threshold values and specified actions. The evaluated decision settings in this paper illustrate this clearly: the heuristics 2A, 2B and 3 can be seen as three different options of monitoring and action plans, while Table 2 specifies the optimized heuristics for the plans and also shows their respective expected costs. Such risk-based optimization of monitoring and actions plans is a considerable leap forward to the current practice, where monitoring and action plans usually are defined based on deterministic analyses, although probabilistic approaches are emerging (e.g., Spross & Gasch, 2019).

9. Conclusion

We have formalized a geotechnical problem as a sequential decision problem, and proposed a heuristic approach to finding optimal strategies. We applied this framework to an embankment preloading problem and highlighted how the decision setting, chosen heuristics and cost model affect the optimal preloading strategies. With this probabilistic framework, the preloading decisions are quantitatively optimized under uncertainty. This framework is not limited to embankment design and construction, but is designed as a decision tool to be extended to a vast range of geotechnical engineering applications, especially those to which the observational method is applied.

Appendix A. Cross entropy optimization algorithm

Algorithm 1 describes the steps of the CE method used for the optimization of the heuristic parameters. The algorithm also applies a smoothing operation, which is not described here, to prevent convergence to local minima (refer to Kroese et al. (2006) for more details). The optimal cost is obtained with Equation (14) evaluated in $\mathcal{S}(\mathbf{w}^*)$.

The sampling density is here chosen as a truncated normal for positive (or negative) parameters. For integer parameters, the sampled value is rounded to the nearest integer. The updated distribution parameters $\boldsymbol{\lambda}^*$ of the multivariate truncated normal distribution are the mean and covariance of the elite samples.

The CE samples obtained can also be used to surrogate the expected cost function, for example using Gaussian process regression (Bismut et al., 2022).

Algorithm 1: Cross entropy method applied to noisy optimization

input: CE sampling density $P(\cdot|\boldsymbol{\lambda}^*)$, initial sampling distribution parameter $\boldsymbol{\lambda}^*$, number of CE samples per iteration n_{CE}, number of elite samples n_E, number of sample settlement trajectories n_{MC}, maximum number of iterations n_{max}.

$l \leftarrow 0$;
while $l < n_{max}$ **do**
  for $m \leftarrow 1$ **to** n_{CE} **do**
    generate random heuristic parameter values $\mathbf{w}^{(m)}$ from sampling density $P(\cdot|\boldsymbol{\lambda}^*)$;
    generate n_{MC} settlement trajectories and measurement following strategy $\mathcal{S}(\mathbf{w}^{(m)})$;
    evaluate the expected total life-cycle cost q_m with n_{MC} samples (Equation (14));
  end
  sort $(\mathbf{w}^{(1)},..,\mathbf{w}^{(n_{CE})})$ in increasing order of q_m;
  fit the distribution parameter $\boldsymbol{\lambda}^*$ to the n_E elite samples;
  $l \leftarrow l+1$;
end
$\mathbf{w}^* \leftarrow$ mean of $P(\cdot|\boldsymbol{\lambda}^*)$;
return $\mathbf{w}^*$

Acknowledgments

Johan Spross' work was supported equally by the Swedish Transport Administration [grant no. TRV 2020/48425] and Formas [grant no. 2018-01017]. The research was conducted without involvement of the funding sources.

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