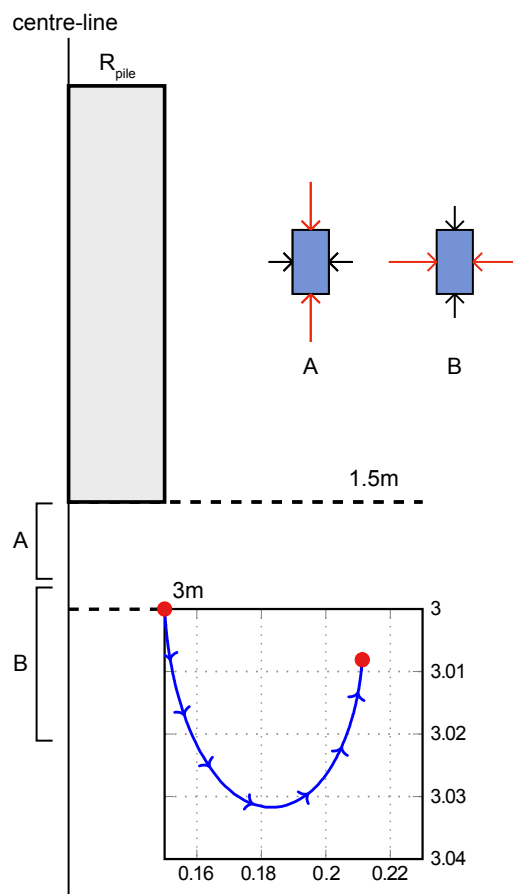




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Long-term benefits for axially loaded deep foundations in clay.

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TECHNICAL REPORT IN GEOTECHNICAL ENGINEERING

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ABSTRACT

A multitude of mechanisms will affect the evolution of bearing capacity and pile head stiffness over time, each with their respective time scale. Most of the processes can be linked to the pile installation stage. Pile installation, and the far less common, pile extraction will alter the soil surrounding the pile. As a result there is a change in the mechanical properties of the soil that will influence the subsequent pile response. Not only the bearing capacity or the initial stiffness response are affected, also the long-term properties will be altered, as the soil surrounding the pile keeps evolving. These long-term mechanisms include the dissipation of excess pore pressures from pile installation and the creep in the soil. For the case considered here, soft sensitive clays, additional negative skin friction from a settling soil profile will contribute to the pile load. Finally, additional material properties such as the initial structure will degrade during installation and partly recover due to thixotropy. The current report presents a concise approach combining the strain path method and an advanced creep model to incorporate the most significant mechanisms to arrive at an assessment of the change of pile response over time. The presented results demonstrate that the degree of remoulding of the soil during the pile installation stage is closely linked to the subsequent pile response, and for certain conditions the positive increase in stiffness and capacity over time. In the current predictions up to 20% increase in undrained shear strength in the clay adjacent to the pile is calculated after a service life of 30 years at 60% of the short-term bearing capacity. Furthermore, creep rupture will be prevented as long as the service load remains below 80% of the short-term bearing capacity after installation and pile setup. When complemented with experimental data, the techniques developed enable a more complete method to assess the geotechnical feasibility of re-using pile foundations.

Keywords: pile installation, time effects, strain path method, data interpretation

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1 Introduction

1.1 Background

In some cases such as densely populated urban areas, offshore, or busy line infrastructure, re-use of existing pile foundations becomes an attractive alternative. The most likely scenario is to mobilise more capacity of an existing pile foundation, so that the service of the super structure can be extended, e.g. increasing the axle loads on line infrastructure. Alternatively, re-use of only the pile foundation might be considered when the structure on top is replaced, and pile extraction or additional pile driving is prohibited due to proximity to adjacent structures. In both scenarios the most important question is the magnitude of the potential additional capacity that can be mobilised after the piles have been loaded for decades. The research question therefore is if there is ‘free capacity’ to be gained due to changing hydromechanical soil properties over time and/or improved understanding/modelling of the mechanisms that reduce conservatism in setting the design load.

In order to understand the long-term response of pile foundations, the complete loading history needs to be considered. In addition to the geological and stress history of the soil, the behaviour of the pile-soil system is directly related to the installation methods, set-up period and loading conditions. Multiple stages from installation to loading can be summarised in a *pile cycle* (Figure 1.1) discussed in the following. Proper understanding of the underlying soil behaviour during the pile cycle will potentially improve the design methods (Randolph and Gourvenec 2011; Karlsrud 2012). Details for this cycle are presented with a focus on pre-fabricated full displacement piles in soft clays. A full description of this pile cycle is elaborated in Yannie (2016), however the main findings are iterated in the following:

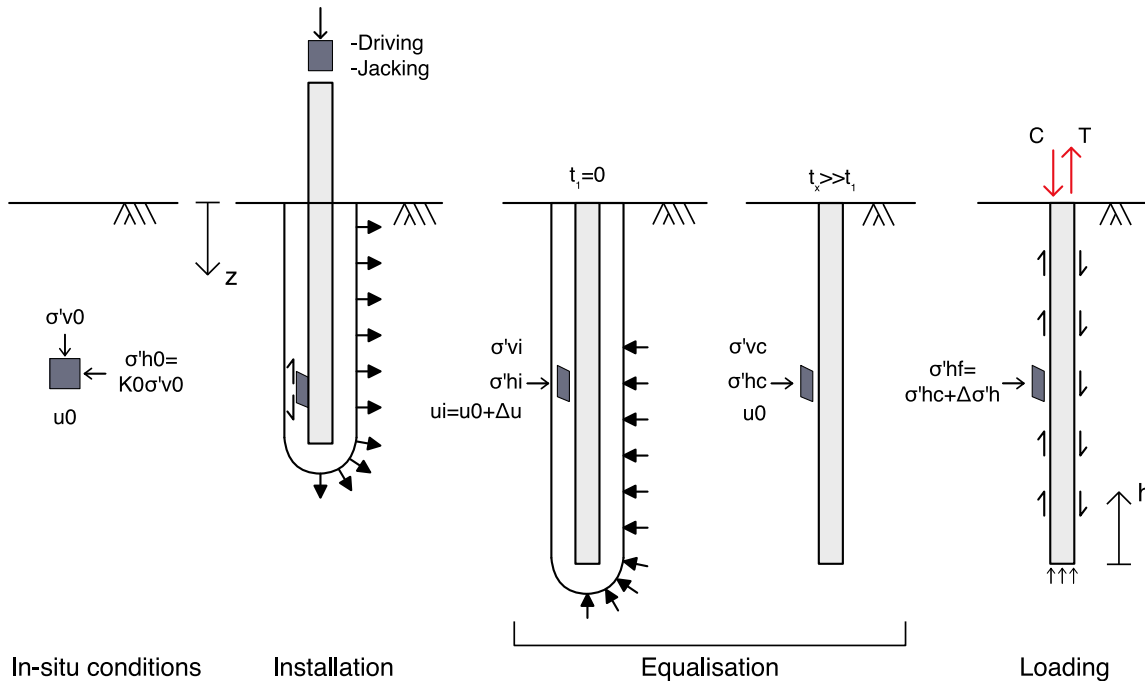


Figure 1.1: *Illustration of the pile cycle (adapted from Randolph and Gourvenec (2011)). 0 = initial conditions, i = installation, c = after equalisation, f = at failure.*

Pile Installation

The first step in the pile cycle is the *installation* of the pile elements in the soil (Fig. 1.1). Installation of displacement piles in soft clays results in a complex kinematic process that generates large distortions in the soil around the pile. Consequently, the soil properties and stress conditions adjacent to the pile shaft will change significantly. This new state will govern the future response of the pile (Lehane and Jardine 1994). Numerous experimental investigations have been reported on pile installation in soft soils, as summarised for example by Hunt (2000), and Karlsrud (2012). Based on these and many other investigations, analytical and numerical models have been developed to capture the kinematics of pile installation using the cavity expansion method CEM (e.g. Randolph et al. 1979) or the strain path method SPM (e.g. Baligh 1985). Even more advanced techniques using large deformation Finite Element methods have been considered, but these are still in the research stage (e.g. Dijkstra et al. 2011). All these methods still need validation of the soil kinematics. Measurement of those are scarce, i.e. only the experiments reported by Ottolini et al. (2014) captured the kinematics and pore pressure fields in a centrifuge test. The latter illustrate that for clays the constant volume assumption is a reasonable approximation for further modelling. Hence there is renewed interest in the Strain Path Method in the current research.

Pile set-up

The *set-up* period is the time when the bearing capacity and initial stiffness recovers due to the equalisation of excess pore water pressures resulting from the pile driving (increase of effective stresses) and ageing in the clay. The latter effect is related to creep and thixotropy (Augustesen et al. 2006).

As the clay consolidates, the excess pore water pressure dissipates, the effective stress increases and the void ratio decreases. The equalisation time depends on the hydraulic conductivity and stiffness of the clay. Remoulded high sensitive clays can have consolidation coefficients on the order of 0.1 to 0.01 of natural undisturbed conditions (Zeevaert 1983). Experience in Gothenburg clay show equalisation times ranging from 3 to 6 months (Fellenius 1972). The reduction in void ratio will translate into an increase in the undrained shear strength of the clay next to the shaft. The latter has also been observed in the field (Roy et al. 1981; Zeevaert 1983; Karlsrud and Haugen 1985)

An additional mechanism for recovery in the apparent undrained shear strength and stiffness has been observed during a rest period without change in water content or effective stress after intense distortion (Seng and Tanaka 2012). In colloid science this is known as thixotropy. For example Mewis and Wagner (2009) define this phenomenon in terms of the colloid micro-structure (i.e. fabric in soils), where the latter will break down into separated flocs when sheared, decreasing in size as the strain rate increases. The micro-structure will recover when the strain rate decreases and recovers further during rest conditions. These findings are for a suspension of colloids, and in soils the findings are not as clear. Yet, after some time during the pile set-up stage, the clay micro-structure will reach a new equilibrium state, after which thixotropic effects will be minimal compared to other processes, such as creep (Seng and Tanaka 2012). Although the structure is partly recovered, the weak cementation is not. Hence, the conservative assumption is that the intact shear strength is never fully recovered due to thixotropy alone.

Pile loading

It is well known from experimental and theoretical considerations that the bearing capacity of a pile is directly proportional to the normal effective stress and interface friction angle at the pile-soil interface (i.e. Coulomb friction) (Randolph and Gourvenec 2011; Lehane and Jardine 1994). For floating piles the load applied at the pile head will mainly transfer as shear to the surrounding soil. Randolph and Wroth (1978) described this transfer mechanism as concentric cylinders in shear around the pile shaft. This will of course increase the stress state in the soil, where the exact stress path is somewhat more complex due to principal stress rotations and the rate dependence of the soil. During pile loading, the observed response is a combination of soil and pile element behaviour. The pile bearing capacity and settlement are a function of the effective stresses and soil properties at the pile shaft, with the load distribution along the pile length depending on the pile stiffness, shaft area and end-bearing capacity.

Long-term pile loading

Research on the long-term performance of piles is scarce. The majority of information available focuses on: (1) the load from negative skin friction due to subsidence of the surrounding soil and (2) on the increase of bearing capacity after installation. A third important, but less studied problem, is (3) the long-term settlement of the pile element under constant load. This settlement occurs due to the primary and secondary compression of the soil adjacent to the pile shaft under shear loading. Here we will mainly focus on scenario (2).

During the equalisation of excess pore water pressures there is a regain in effective stresses in the soil. At the same time, ageing and creep effects occur. It is not yet clear which mechanisms take place during the ageing of piles in clay. A plausible hypothesis is that circumferential arching develops during installation, relaxing with time due to creep (Augustesen 2006). Other possible explanations are the creation of new bonds and rearrangement of the fabric due to thixotropy effects. However, these hypotheses are difficult to validate. Instead, empirical relations (often called *time functions*) are used to quantify the increase in bearing capacity with time due to these ageing effects. Recently, Karlsrud et al. (2014) investigated the effects of sustained loading in the increase of bearing capacity with time. In their study, piles were loaded 6 months after installation with a sustained load ratio of $Q/Q_{ult} = 0.6$. The piles were loaded to failure 18 months after pre-loading. The sustained load enhanced the shaft resistance compared to first time load test of non-loaded piles with the same set-up time. An additional increase of 10 to 20% was measured for slightly over-consolidated medium to high plasticity clays. Karlsrud et al. (2014) postulated that the results support the plausible theory of increase of effective stresses at the shaft due to creep relaxation of arching effects.

These findings give a strong indication that by including the pile installation stage in an effective stress analysis of the pile response, using a model formulation for the soil that incorporates creep, a substantial improvement can be made in assessing the gain in bearing capacity over time. This approach, in the end, will be based on sound understanding of soil mechanics without the need to resort to empirical relations.

1.2 Aim

The aim of the project is to combine the existing experimental evidence on pile installation and its effects over time with the latest insights in modelling soft soil behaviour to arrive at

a rigorous method to predict the pile response over time. The focus of the study will be on explicitly incorporating the pile installation stage in the quantification of the increase in shaft resistance over time.

1.3 Objectives

- Extraction of key mechanism relevant to pile setup from experimental data
- Implementation of the strain path method & creep model
- Quantifying gains in bearing capacity for a relevant Gothenburg case

1.4 Limitations

- The study only focuses on axially loaded piles.
- Only main mechanisms in clay will be incorporated (pile installation, consolidation, creep).
- Study is limited to the Gothenburg Central Station case for tension loaded piles.

2 Modelling increase in bearing capacity

2.1 Introduction

Time-dependent effects for foundation elements such as piles are often studied separately from the soil behaviour. This leads to a multitude of interpretations on the underlying mechanisms that underpin the observed pile head response over time. Alternatively the mechanisms are simply ignored altogether and empirical relations to quantify the increase in bearing capacity with time due to pile setup effects are formulated instead. These are derived from pile tests where the same pile is loaded several times within a certain time period (staged testing) or different piles are loaded once after certain time (unstaged testing). The most well known relation is the one proposed by Skov and Denver (1988) which is not giving any additional insight in the mechanisms.

In this research project we use an alternative approach where we limit ourselves to known concepts based on a basic understanding of soil behaviour under the particular constraints of pile loading.

2.2 Capturing the relevant soil response

Soft sensitive soils have specific features that need to be incorporated in the constitutive model in order to capture the appropriate mechanical response. Here, we follow the work of Karstunen and co-workers who developed the SCLAY1(s) family of models that incorporate anisotropy (strength/stiffness); destructuration (bond degradation) and creep. The rationale and model details have been discussed in length elsewhere (e.g. Wheeler et al. 2003; Karstunen et al. 2005; Sivasithamparam et al. 2015), however, two main model features and their relevance to the long-term pile response are further discussed below. Those are destructuration and creep (sometimes called rate-dependency). The 3D stress model is implemented in a single element strain driver.

2.2.1 Destructuration

The (Creep)-SCLAY1s models have a third hardening rule that describes the degradation of bonding χ as function of plastic volumetric and deviatoric strains (see Equation (2.1)). It is closely linked to the first isotropic hardening rule in the critical state models. The relative size of the intrinsic yield surface, i.e. soil without any structure left, is related to the natural yield surface of the intact soil with a quantity χ that reflects the amount of bonding (see Equation (2.2)). Therefore, the natural yield surface (which size is governed by the initial in-situ stress state of intact soil) will shrink and the intrinsic surface (which describes the size of the yield envelope of a material without bonds left) will expand or shrink (depending on the flow rule) until there is no bonding left and they have the same size.

$$d\chi = -\xi \chi (|d\varepsilon_v^p| + \xi_d |d\varepsilon_d^p|) \quad (2.1)$$

$$p'_m = (1 + \chi)p'_{mi} \quad (2.2)$$

where $d\varepsilon_v^p$ and $d\varepsilon_d^p$ are the volumetric and deviatoric plastic strains, respectively, ξ is the parameter controlling the absolute rate of bond degradation and ξ_d is the parameter controlling the relative bond degradation between the volumetric and deviatoric component of plastic strain.

It becomes apparent that this formulation only incorporates the reduction of strength and stiffness due to plastic strains. In laymen terms it reflects the ‘degree’ of remoulding. In case we explicitly model pile installation, we will therefore arrive at a more representative state. Although the current hardening law incorporates the increase in (intrinsic) strength as function of time due to creep effects, it doesn’t explicitly model thixotropic effects. The latter could be an probable addition to include the time dependent effects from thixotropy in the formulation. However, thixotropy is seen as a mechanism of secondary importance compared to the dissipation of pore pressures (hence effective stress build up) and creep.

2.2.2 Creep

In addition to anisotropy and structure, soft clays also exhibit viscous behaviour (rate dependency/creep). This additional property incorporates the time dimension in the material description. Creep-SCLAY1s is an elasto-viscoplastic constitutive model capable of modelling rate dependency. It is similar to the SCLAY1s in the manner anisotropy and structure are formulated. In addition, it is a special extended over-stress model that uses a generalised empirical formulation obtained from one dimensional observations to model the rate dependent behaviour of the soil. As a result the additional viscous parameters can be readily obtained from standard incremental loading oedometer tests. The full theoretical formulations can be found in Sivasithamparam et al. (2015).

It is important to mention that the mechanism in the soil adjacent to the pile shaft result mainly in distortion, i.e. shear strain. This is something the model automatically resolves without the need for model calibration with data from special deviatoric creep tests (these are however very useful for the model development and validation).

Including a creep formulation in the modelling response will enable to capture the on-going time-dependent gain in pile capacity beyond the effects of thixotropy (days) and pore pressure dissipation (months), as observed in the field tests (Karlsrud et al. 2014).

2.3 Including pile installation

Installation of displacement piles, such as the pre-cast concrete piles common in Sweden, introduce large distortions in the soil. Displacement pile installation is an active research area with numerous contributions. Numerical modelling of this process using Finite Elements is complex. The large deformations during this process require special attention ranging from re-meshing strategies, to advanced Arbitrary Lagrangian Eulerian solvers or even mesh-free approaches using the Material Point Method. Regardless of the method used, the combination of an advanced numerical framework, the inclusion of couple porewater consolidation equations and an advanced non-linear model for the soil is still not fully resorted. Hence, here we exploit the constant volume conditions obeying an associated flow rule of a saturated clay. The kinematics, as obtained from physical modelling tests on the installation of a pile in clay, reinforce this observation (Ottolini et al. 2014).

2.3.1 Strain Path Method (SPM)

The Strain Path Method (SPM) was developed based on field and laboratory observations of installation of rigid objects in soils (Baligh 1985). This method provides an analytical approximation of the installation effects of displacement piles in soft clays. The advantages of this method compared to other analytical approaches (e.g. Cavity Expansion Method) is that vertical soil displacements are considered and the pile penetration is approximated under steady state. Baligh (1985) presented a full description of the method and its application to piles, soil samplers and in-situ testing devices. The assumption behind SPM is that the deformations and strains for deep penetration problems are independent of the soil behaviour due to the dominating kinematic constraints in this process. The method assumes that penetration occurs under quasi-static steady state conditions, in incompressible, isotropic, homogeneous, non-viscous and rate independent soil under isotropic stress conditions and with no roughness at the pile-soil interface. Therefore, the penetration process reduces to a flow problem where the soil particles flow around a rigid penetrating object.

The implementation considered here, is a further development that uses the strain paths resulting from the analysis in a strain driver to arrive at the expected change in soil properties after pile installation. Furthermore it starts from the work initiated by Sagaseta et al. (1997) whom incorporated the ground surface effects together with the deep penetration solutions presented by Baligh (1985). This results in the Shallow Strain Path Method (SSPM). As opposed to the SPM, the SSPM does not have a reference system fixed to the penetrating object, but instead considers a transient source moving from the free surface into deep layers. Therefore, the penetration process is no longer a steady state process. As the source penetrates into deeper layers, the solution approximates that from the SPM. In other words, the surface effects are not longer dominant. Therefore, the sink and shear traction in their approach can be neglected. By numerical integration of Sagaseta et al. (1997), the velocity and the strain rate field given by the moving source, the soil deformations and strain paths can be obtained in time (or equivalent penetration depth). The position of a soil particle in Cartesian coordinates is given by Equation (2.3).

$$\begin{aligned} x(h) &= x_0 + \int_0^h v_x(x, z, h) \frac{1}{U} dh \\ z(h) &= z_0 + \int_0^h v_z(x, z, h) \frac{1}{U} dh \end{aligned} \tag{2.3}$$

where x_0 and z_0 are the initial soil particle coordinates, v_x and v_z are the velocities in the x and z direction induced by the penetrating source from 0 to h and U is the moving speed of the source. The x and z coordinates in the integrand part change as the source moves from 0 to h , corresponding to a large strains solution (updated geometry). If x and z are not updated and taken as x_0 and z_0 , the problem is solved assuming small strains.

Here, the velocity field of interest is that of the simple pile case. In this approach a cylindrical coordinate system is used, with $x = r$ and $z = z$. The velocities and strain rates in each direction are given by Equations (2.4) and (2.5).

$$\begin{aligned}
v_r(r, z, h) &= \frac{UR^2}{4} \frac{r}{r_1^3} \\
v_z(r, z, h) &= \frac{UR^2}{4} \frac{z - h}{r_1^3}
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
\dot{\epsilon}_{rr} &= -\frac{UR^2}{4} \frac{1}{r_1^3} \left(1 - 3\frac{r^2}{r_1^2} \right) \\
\dot{\epsilon}_{\theta\theta} &= -\frac{UR^2}{4} \frac{1}{r_1^3} \\
\dot{\epsilon}_{zz} &= -(\dot{\epsilon}_{rr} + \dot{\epsilon}_{\theta\theta}) \\
\dot{\epsilon}_{rz} &= \frac{UR^2}{4} \frac{1}{r_1^3} \frac{3r(z - h)}{r_1^2}
\end{aligned} \tag{2.5}$$

where r_1 is the distance from a point in the space $P(r, z)$ to the current location of the source $S(0, h)$, as calculated in Equation (2.6).

$$r_1 = \sqrt{r^2 + (z - h)^2} \tag{2.6}$$

2.3.2 Strains

Equations (2.4) and (2.5) were numerically integrated in a 2D domain by using the equation-based modelling in the commercial software COMSOL Multiphysics. The displacements and strains were obtained from a radial cross section far below the free surface and far above the pile base. For deep penetration, all soil elements in the radial direction will experience the same strain paths (steady state). In this way, the strain paths can be determined for a given depth and all other variables can be normalised with the initial effective vertical stress at that point. A total of 201 discrete nodes were used to extract the strain paths. These were located at a spacing of 0.05:0.01:1.5 – 1.6:0.1:5 – 5.5:0.5:15 meters from the centre line of penetration. The solution precision for the strain path in each node depends on the mesh size and time step in the numerical integration of the velocity field in COMSOL.

2.3.3 Stresses

The effective stresses for each discrete node were calculated using the strain paths obtained in COMSOL Multiphysics and the SCLAY1s soil constitutive model implemented in the single element strain driver called VAMP (graphical interface, sensitivity and optimisation software for incrementalDRIVER) (Gudehus et al. 2008; Gras et al. In press). A MATLAB script was used to communicate with VAMP and loop all 201 nodes. The elastoplastic SCLAY1s model proved to be more stable than the Creep-SCLAY1s model to simulate the installation processes in the single element strain driver.

The initial stress conditions are taken for the middle section of the pile, 15 meters below the ground surface. At this point $\sigma'_{v0} = 90$ kPa and $\sigma'_{h0} = 54$ kPa (assuming $K_0 = 0.6$). The soil model parameters are given in Table 2.1. Details of the parameter determination procedures can be found in Wheeler et al. (2003), Karstunen et al. (2005) and Gras et al. (In press).

Table 2.1: Parameters for SCLAY1s at 15m depth, Marieholm.

κ^*	λ_i^*	λ^{*1}	v'	M_c	α_0	ω	ω_d	χ_0	ξ	ξ_d	OCR	e_0
0.015	0.1	0.25	0.2	1.64	0.5	150	1	14	9	0.4	1.3	2

¹ Used instead of λ_i^* when the effect of bonding χ is not considered.

2.3.4 SPM excess pore water pressure

Given the effective stresses obtained from the strain paths and the soil constitutive model at the discrete soil nodes, the excess pore water pressure is calculated from the equilibrium conditions of total stresses. According to Baligh (1985) during undrained penetration, the change in total stress is governed by the equilibrium Equation (2.7) (in a cartesian frame and $i, j = 1$ to 3). Here x_i are the coordinates of a material point and repeated indices imply summation over 1, 2 and 3.

$$\begin{aligned} \frac{\partial \sigma_{ij}}{\partial x_i} &= 0 \\ x_i &= (x_1, x_2, x_3) \end{aligned} \quad (2.7)$$

The total stress is the sum of effective stress and pore water pressure as stated in Equation (2.8) ($\delta_{ij} =$ Kronecker's delta). Hence, the pore water pressure is obtained from the equilibrium Equation (2.9).

$$\sigma_{ij} = \sigma'_{ij} + u\delta_{ij} \quad (2.8)$$

$$\begin{aligned} \frac{\partial u}{\partial x_i} &= -\frac{\partial \sigma'_{ij}}{\partial x_i} \\ g &= -\frac{\partial \sigma'_{ij}}{\partial x_i} \end{aligned} \quad (2.9)$$

In a 2D axisymmetric problem, g in the above equation (multiplied by -1) is given by the equilibrium Equation (2.10), for the radial and vertical direction respectively. However, the solution for Δu will depend on the integration path as the constitutive model does not correspond to the assumptions made for the strain paths and the pore pressure field is not in equilibrium in all directions. Aubeny (1992) showed that at the shaft (far above the base), Δu is best approximated by radial integration. Below the base level the vertical integration works best. The integration path dependency is improved by taking the divergence of Equation (2.9) and solving numerically the resulting Poisson's Equation (2.11) (Baligh 1985; Aubeny 1992).

$$\begin{aligned} -\frac{\partial u}{\partial r} = -g_r &= \frac{\partial \sigma'_{rr}}{\partial r} + \frac{\partial \sigma'_{rz}}{\partial z} + \frac{\sigma'_{rr} - \sigma'_{\theta\theta}}{r} \\ -\frac{\partial u}{\partial z} = -g_z &= \frac{\partial \sigma'_{zz}}{\partial z} + \frac{\partial \sigma'_{rz}}{\partial r} + \frac{\sigma'_{rz}}{r} \end{aligned} \quad (2.10)$$

$$\nabla^2 u = -\nabla g = -q \quad (2.11)$$

For deep penetration problems, strains along the pile shaft far above the tip tend to reach a steady-state (with every element in the radial direction experiencing the same strain and stress path at all depths). Therefore, the problem is reduced to solve Equation (2.9) in 1D (i.e. radial direction). For this condition Equation (2.9) and (2.10) become:

$$\begin{aligned} g_r &= - \left(\frac{\partial \sigma'_{rr}}{\partial r} + \frac{\sigma'_{rr} - \sigma'_{\theta\theta}}{r} \right) \\ u &= \int g_r \, dr = \int - \left(\frac{\partial \sigma'_{rr}}{\partial r} + \frac{\sigma'_{rr} - \sigma'_{\theta\theta}}{r} \right) \, dr \end{aligned} \quad (2.12)$$

In this 1D case, the radial integration is performed starting from the far field towards the pile shaft located at R from the pile centre line (Teh 1987; Aubeny 2016). The integration can be solved numerically with a trapezoidal rule as given in Equation (2.13), starting with a node i in the far field and moving to the next node $i + 1$ closer to the pile shaft in the same radial line j . In this integration scheme, the cavity stress ($\sigma'_{rr} - \sigma'_{\theta\theta}$) will be the most sensitive stress component to numerical errors (Aubeny 2016).

$$\begin{aligned} u &= \int_{\infty}^{R_{pile}} - \left(\frac{\partial \sigma'_{rr}}{\partial r} + \frac{\sigma'_{rr} - \sigma'_{\theta\theta}}{r} \right) \, dr \\ u_{i+1,j} &= u_{i,j} - \left(\frac{du}{dr} \right)_M \Delta r_i \end{aligned} \quad (2.13)$$

where u is the pore pressure at a node i , $(du/dr)_M$ is the pore pressure gradient at the middle point between node i and $i + 1$ and Δr_i is the spacing between the integration nodes.

The far field boundary is given by a Dirichlet condition where $\Delta u = 0$. The boundary at the pile shaft, however, is not known a priori. What is known is that at this latter location $\Delta u \neq 0$ and the pore pressure gradient is not necessary $\partial u / \partial n = 0$ (i.e. Neumann condition indicating no pore pressure gradients normal to the pile shaft). As the integration is performed from the far field side, the boundary at the pile shaft will emerge automatically and there is no need to specify it beforehand.

2.3.5 SPM equalisation of Δu

Far above the pile base, the equalisation of the excess pore water pressures can be approximated by 1D radial consolidation (Whittle 1987). In this case, the effective stresses, installation pore water pressure and state variables from the SPM discrete elements must be transferred to the 1D mesh. During pile installation, the shear component in the rz plane can be significant. However, Whittle (1987) stated that this do not affect the consolidation process and can be neglected.

Aubeny (1992) observed that for the 2D consolidation case, the installation total stress field do not necessarily satisfy internal equilibrium and might be incompatible with the 2D boundary conditions. The main incompatibility arises from the shear stress σ'_{rz} at the pile

shaft and ground surface. One of the solutions considered here is to add an initial undrained step in order to allow the soil to equilibrate with the boundary conditions.

The installation stresses and state variables obtained from the SPM and the SCLAY1s model were passed on to the Creep-SCLAY1s model. Then, the single element was loaded to the set-up stresses in a period of 90 days simulating the consolidation time. The K_c value used was 0.4 for the slightly over-consolidated Marieholm clay. Further details on obtaining the stress state after pile installation, pore pressure dissipation (pile setup) are found in Yannie (2016).

2.3.6 Pile load test

A representative point near the pile shaft is further investigated for its long-term response using the Creep-SCLAY1s model. Estimates for the deviatoric creep rate after pile installation and set-up in a disturbed soil profile are obtained by application of a drained shear loading path similar to that of a Direct Simple Shear (DSS) test at a single stress point. This represents the pile response under working loads.

The creep rates are studied for different degrees of mobilisation in respect to the undrained peak shear strength of the element test (τ_{rz}^{peak}). This peak strength represents the undrained strength directly after pile installation and set-up. In addition, different boundary conditions for the DSS test are considered (see Figure 2.1). These boundary conditions (BC) are described below:

- BC1: no volume change is allowed in the soil element. Undrained BC.
- BC2: the radial (normal) stress perpendicular to the shear load is kept constant. The soil element will change in volume with the radial strains. Drained BC.
- BC3: the vertical and circumferential stresses are kept constant. The soil element will change in volume with the vertical and circumferential strains. Drained BC.
- BC4: all normal stresses are kept constant. Therefore, no relaxation will take place in the soil element with the volume changes. Drained BC.

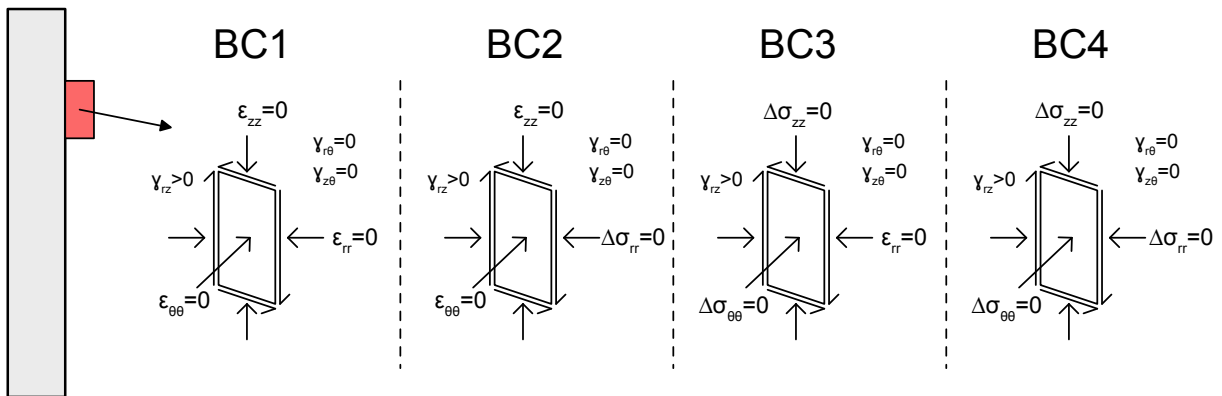


Figure 2.1: *Different boundary conditions for DSS test.*

The stresses at the pile mid section are used for calculating the set-up stresses, with $\sigma'_{v0} = 90$ kPa. The model parameters used are presented in Table 2.2. Note that here the parameters

related to the initial and evolution of soil structure (χ_0, ξ, ξ_d) are now 0 as the pile installation simulations resulted in χ_0 tending to 0 adjacent to the pile. Therefore, the corresponding model parameters are set to 0 too.

Table 2.2: Parameters for Creep-SCLAY1s at the pile shaft following pile installation (15m depth, Marieholm).

κ^*	λ_i^*	λ^*	v'	M_c	M_e	α_0	ω	ω_d	χ_0	ξ	ξ_d	OCR	e_0	μ^*_i	t_{ref}
0.015	0.1	-	0.2	1.64	1.14	0.55	50	1	0	0	0	1	2	0.003	1

3 Results

3.1 Introduction

The results of the soil response after installation and pile-setup will be calculated on the integration point level using a single element driver and the Creep-SCLAY1s model. This is a reasonable approach for long piles where the shaft resistance dominates the total bearing capacity. Furthermore, the failure mechanism is assumed to occur in the soil close to the pile, but not directly on the interface.

3.2 Shear strength directly after installation

The short-term undrained and drained shear strength were calculated after initialisation of the set-up stresses, i.e. the state after pile installation and dissipation of the excess pore pressures. Three different rates are used in both test types in order to study the effect of loading rate. The applied loading rates are summarised in Table 3.1. The results from the undrained shear test at 4% / h and the drained shear test at 4% / 10 d are used as reference for the degree of load mobilisation (the design load) in the long-term tests.

Table 3.1: Short-term DSS loading rates.

Undrained	Drained
4 % / h	1.176 % / h
4 % / d	4 % / 10 d
4 % / 10 d	–

All test results are presented in Figure 3.1. Note that for the drained tests two different boundary conditions are compared, namely BC2 and BC3.

The shear strength in DSS will depend on the Lode angle, with $\theta = 0$ (Doherty and Fahey 2011). The critical state line M is calculated using Equation (??) for the CREEP-SCLAY1S model, giving $M_{DSS} \approx 1.3$, as observed for the undrained and drained test with BC2. For the reference undrained and drained (with BC2) loading rate, the peak strength was $\tau_{rz} = 25$ and $\tau_{rz} = 25.6$ kPa respectively. These peaks are very similar in magnitude despite the different stress paths. For the drained tests with BC3, the radial stress relaxed and failure took place at the compression critical state line $M = 1.64$ and $\tau_{rz} = 27.2$ kPa (comparable to the triaxial compression). Remarkably, the stress path in the $\tau_{rz}-\sigma'_{zz}$ plot are very similar to the trends observed by Lehane and Jardine (1994).

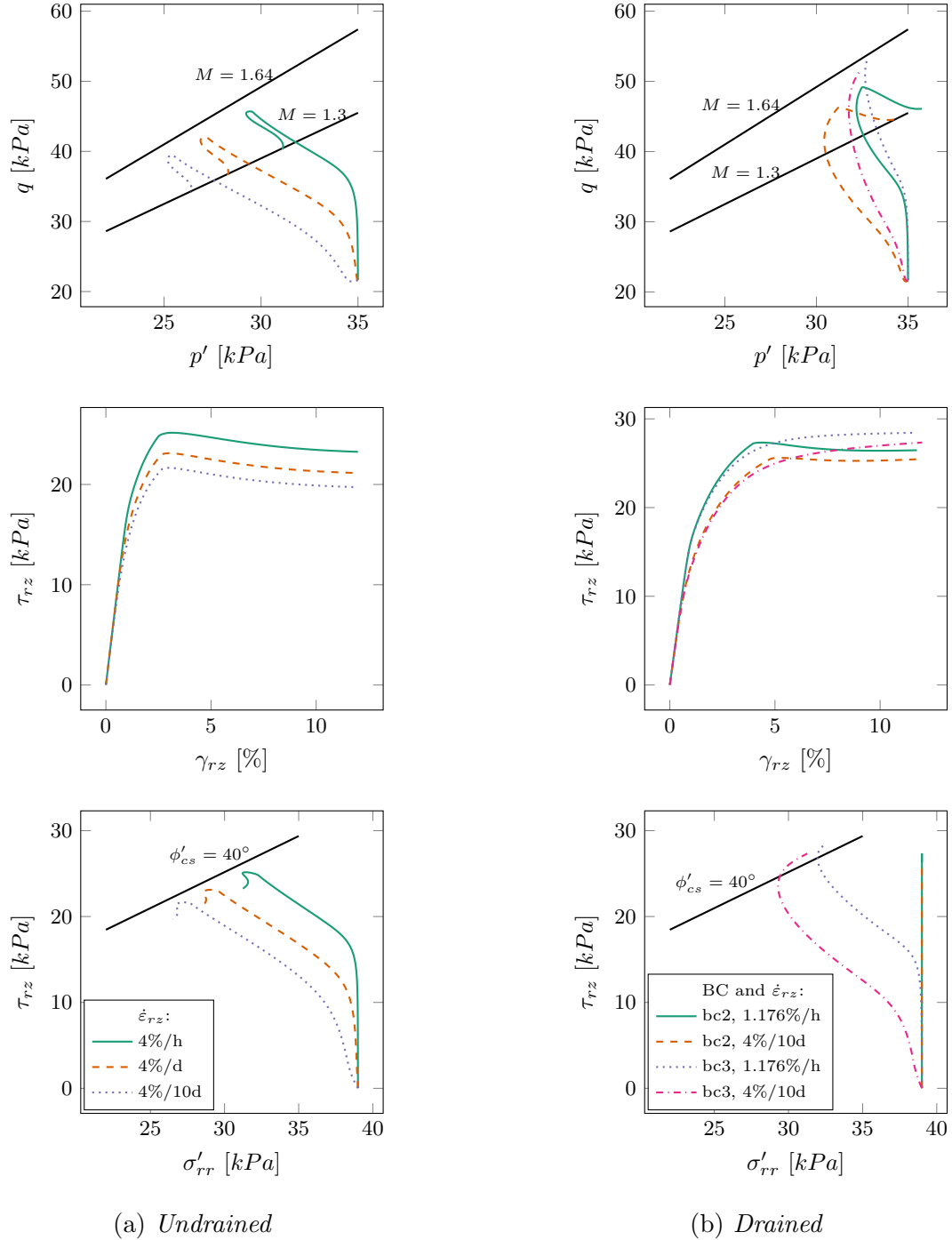


Figure 3.1: Short-term loading of single element under DSS conditions.

3.3 Long-term pile response under service load

The creep rates for different service loads (or degree of load mobilisation) are calculated by means of long-term holding periods. First the desired shear load is applied in an 1 hour increment. Thereafter the load is kept constant, and the soil is allowed to creep for 365 days. In this analysis, all boundary conditions are compared in order to study the effects of stress relaxation under different kinematic constrains. The results are presented in Figure 3.2 to 3.4.

All Figures plot the stress paths in mean effective stress - deviatoric stress space ($p-q$), effective horizontal stress and shear stress space ($\sigma'_{rr} - \tau_{rz}$) formulation also the shear strain shear stress plot ($\gamma'_{rz} - \tau_{rz}$) and the time - shear strain evolution are plotted ($t - \gamma'_{rz}$).

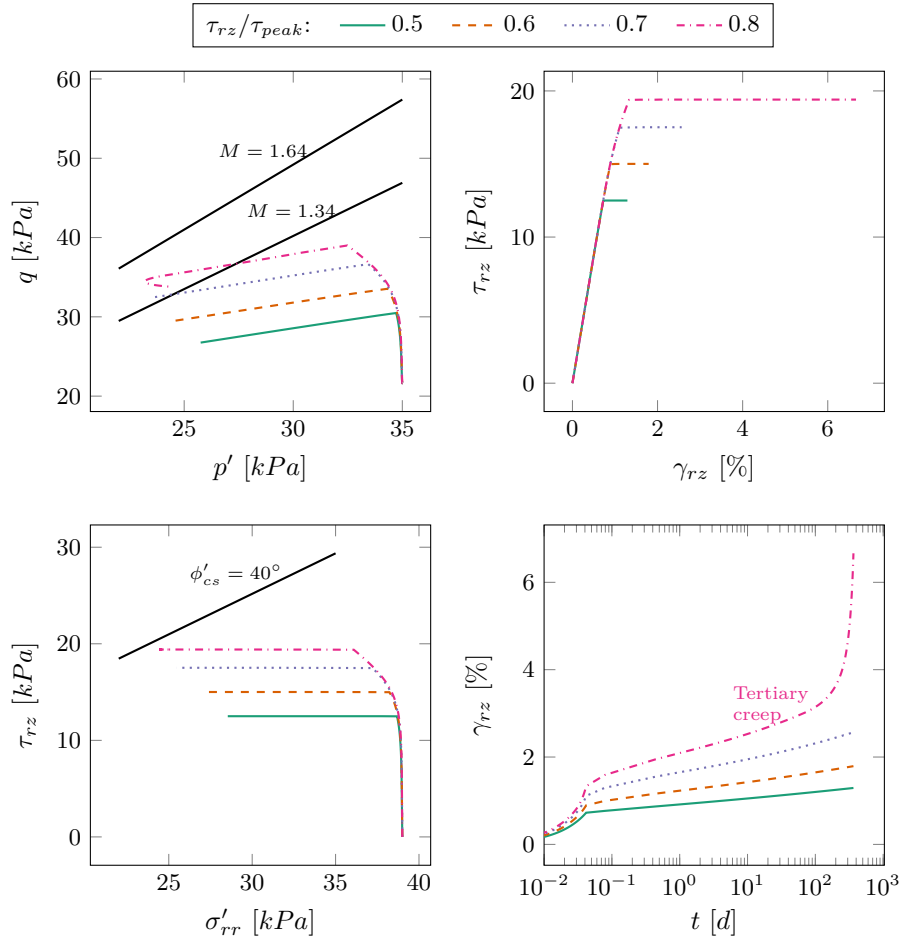


Figure 3.2: Deviatoric creep during simulated service life; Undrained (BC1).

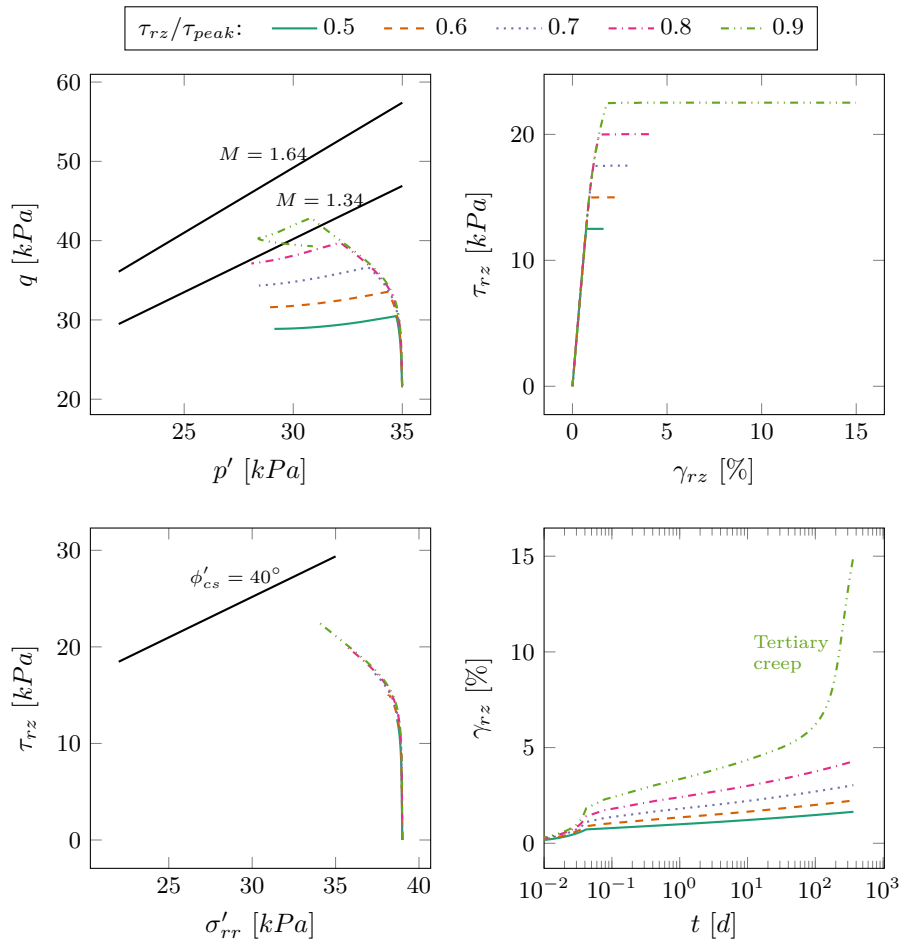


Figure 3.3: *Deviatoric creep during simulated service life; Drained (BC2).*

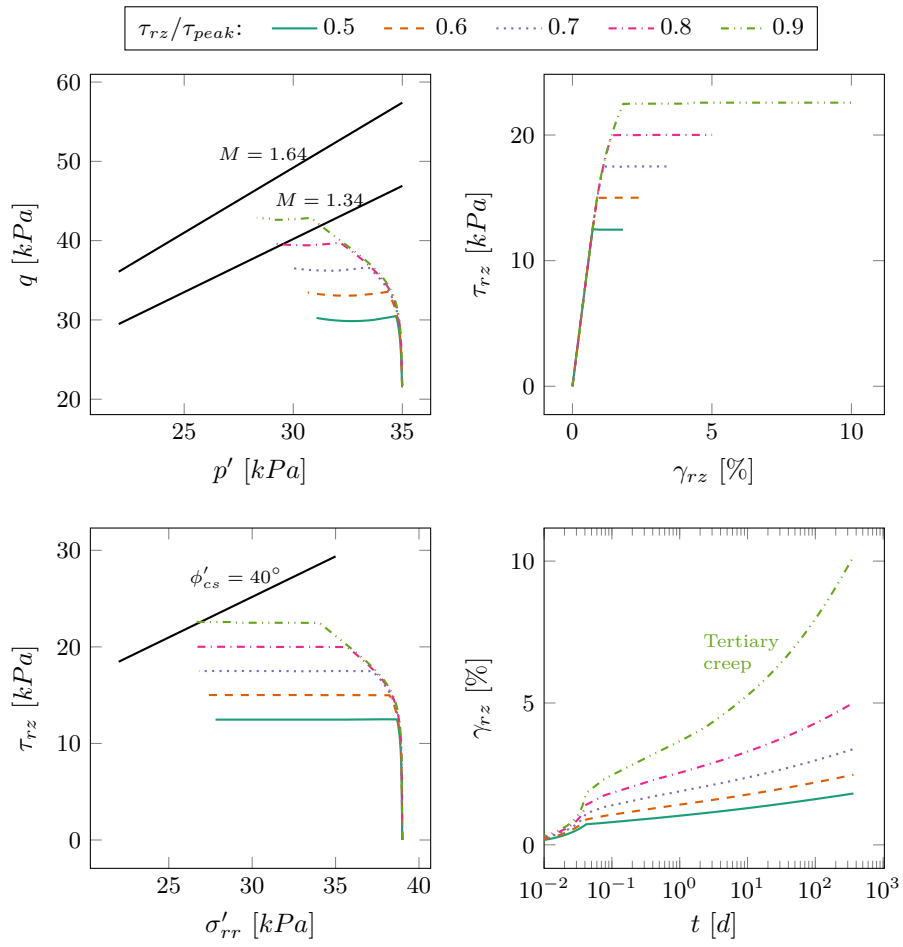


Figure 3.4: *Deviatoric creep during simulated service life; Drained (BC3).*

As can be expected during the creep stages, the accumulated shear strains increased with time and increasing shear stress magnitudes. These approximately follow a linear trend in a semi-logarithmic plot, showing an gradually increasing slope with increasing shear stresses; similar to the results observed in the field tests presented in Yannie (2016).

At mobilised ratios greater than 80% of the shear strength directly after loading, the creep parameter becomes larger than the intrinsic value as the stress approaches the failure envelope. For BC1, BC2 and BC3 a mechanism similar to creep rupture occurred when the soil stresses approximate the maximum stress obliquity given by the critical state friction angle ($\tau/\sigma'_n = \tan(\phi'_{cs})$). At this stage the creep rate accelerates and the soil can not sustain any longer the applied shear load. Especially for the drained BC2 and BC3 this is a bit unusual, perhaps indicating that also for sensitive soil in the field the “rupture” not necessarily is a totally undrained mechanism but partly material softening. The maximum mobilised shear ratio τ/τ_{rz}^{peak} is 0.8 for BC1 and 0.9 for BC2 and BC3.

3.4 Shear strength after long-term service loads

The increase in shear strength after long-term pile loading is further investigated for the most relevant boundary condition (BC3) using aforementioned model parameters and simulation strategies for pile installation, setup and long-term loading.

The applied service load is selected to be 60% of the shear strength directly after pile installation and pile setup, using the single element DSS simulation. This is in line with typical design values in industry and well below the threshold for creep rupture. Note that the load application is 1 day which is slower than the 4%/hr loading rate of the fast test reported above, but faster than a typical construction time of the super structure. Gains in shear strength after 1.5 years and 30 years service life are evaluated. The first time period represents a qualitative verification against the only field tests with service loads for 1.5 years reported in literature, i.e. Karlsrud et al. (2014) who also loaded up to 60% of the failure load. Secondly, a 30 years time period is selected to represent a period that is closer to the economic life time of the structure on top, after which changes in use (hence foundation loads) can be expected.

The results of the simulations are shown in Fig. 3.5. In all cases an increase in the shear strength is predicted. The order of magnitude of the strength increase after 1.5 years of about 12% is in line with the findings of Karlsrud et al. (2014). In the subsequent 28.5 years an additional gain in strength of ca. 6% to a total of 18% compared to the initial situation is predicted. Similar values for the increase in peak and residual strength are found with only limited softening. The latter modelled soil response is not unreasonable, as all structure (bonding) in the intact soil already was destroyed during the pile installation stage. The initial stiffness increases with similar magnitudes as the strength.

It becomes apparent that, for the chosen simplifications and boundary conditions, the relaxation of horizontal contact stress σ_{rr} is the main component for the change in the mean effective stress. In Creep-SCLAY1s modelling terms the change in stress due to creep also continues to grow the normal compression surface, i.e. the soil becomes over-consolidated during creep.

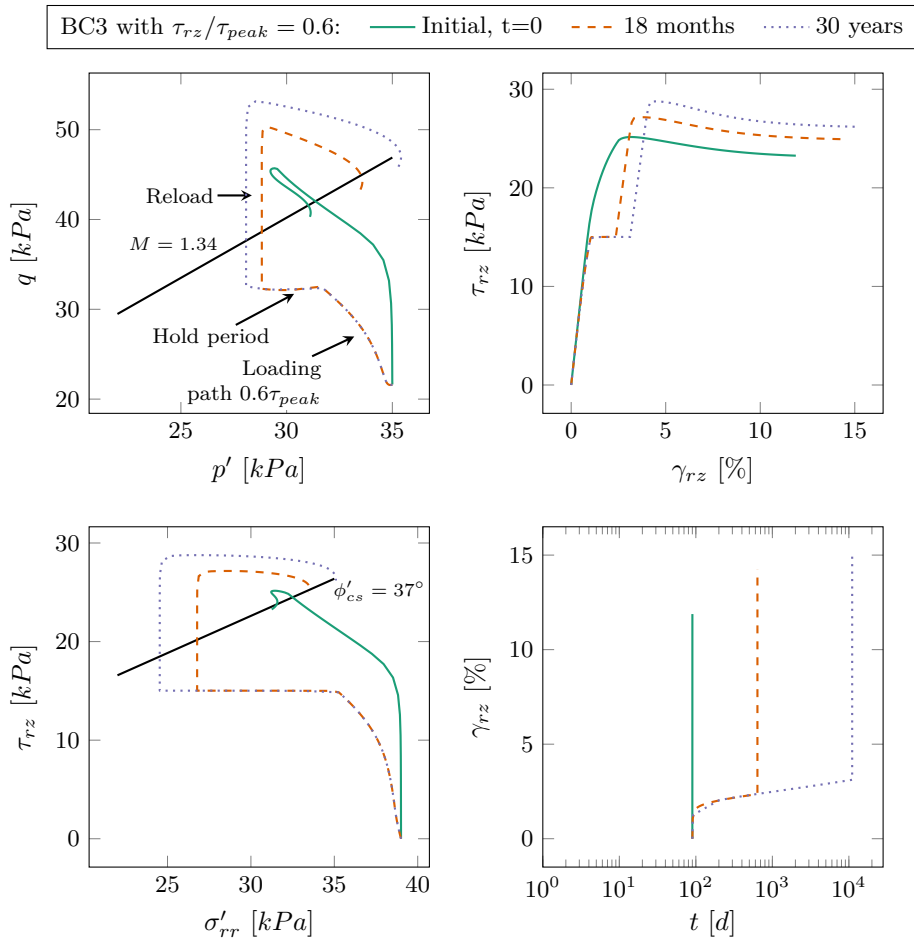


Figure 3.5: Increase in shear strength after 1.5 yrs & 30 yrs. simulated service life at 60% full capacity; Drained (BC3).

4 Conclusions & Recommendations

4.1 Conclusions

The long-term pile response is successfully simulated by combining the Strain Path Method and an advanced model for soft soils that incorporates anisotropy, destructuration and creep. The current implementation, whilst in its infancy, is based on well judged simplifications from physical model tests (constant volume kinematics), boundary conditions during loading (DSS like) and creep rates (full field pile load tests). The results indicate that the advanced time-dependent features can be captured with the required detail. This opens up new avenues to investigate time-dependent effects related to (deep) foundations.

The presented results demonstrate that the degree of remoulding of the soil during the pile installation stage is closely linked to the subsequent pile response, and for certain conditions the positive increase in stiffness and capacity over time. In the current predictions up to 20% increase in the undrained shear strength in the clay adjacent to the pile is calculated after a service life of 30 years loaded at 60% short-term bearing capacity. Furthermore, creep rupture will be prevented, as long the service load remains smaller than 80% of the short-term bearing capacity after installation and pile setup. Ultimately, when complemented with experimental data, the approach adopted leads to more complete assessment of the geotechnical feasibility of re-using pile foundations.

4.2 Recommendations

This research is only a first step towards a full comprehension and modelling capability of the long-term response of pile foundations that include the main governing mechanisms. The following recommendations will help to advance the work that has been initiated in the current research project:

- Study more load combinations
- Further refine the stress equilibration stage in the adopted method.
- Extend the method to a 2D axi-symmetric numerical model where the full pile length is considered and benchmark this against large deformation finite element simulations.
- Collect more reliable data for long-term pile load tests (10 years or more).

5 Discussion

This Chapter will address the detailed response on the follow up questions and feedback from Trafikverket. Response on the remarks from Anders Hallingberg by Jelke Dijkstra is given in italic text below. It aims to place the current work in the right context, as well as helps to pin point questions for follow up research, and implementation areas worth considering.

5.1 Remark 1: Tension vs. compression; load type

First: How strictly are the conclusions related to just tensioned piles? The most applicable area of use would be compression piles. Is it possible to increase the load from either the traffic on a structure (where there is a dynamic factor which correspond to shear strength related to fast loading cycles) or from a future heavier bridge structure (shifting from steel to concrete superstructure).

All of the remarks above are very relevant questions. They will be separately addressed as follows:

- **Tension/compression:** *The results are mainly aimed at quantifying the effects at the pile shaft. To isolate this mechanism tension loads are considered so that the pile base is not influencing the results. There is reason to believe that the response in compression (at the shaft) is very comparable. There are two arguments in favour for this: (1) Existing data from Augustesen and (2) that strictly speaking the soil model used is not optimal in load reversals (i.e. the tension loads are properly predicted, with a model that primarily is designed for monotonic loading). It is recommended to look into this aspect in more detail in future research, though the expectation is that the results will not change that much in case only one load reversal (from installation to pile loading) is considered.*
- **Dynamic factor:** *Dynamic/cyclic effects indeed are of concern. Some of those are already considered in other Chalmers projects from Trafikverket. It turns out to be rather complex, but so far it seems that the creep rate is affected the most (increasing the on-going displacements). It is unclear at this stage if this will also increase the likelihood for creep rupture (not studied yet).*
- **Changing the static load component:** *This is possible, within a certain limit, as a 15%-20% increase in pile capacity (already at 60% service load in a couple of years) is found. So yes, definitely worth exploring. It would, however, be recommended to perform a couple of additional tests to establish if this mechanism is a strength increase beyond original strength of the intact material (at a certain void ratio or water content), or simply a partial recovery of strength after pile installation which never recovers to the level of the intact strength of sensitive clay with weak bonding.*

5.2 Remark 2: Single pile vs. pile group

The report describes the behaviour of a single pile mainly focusing on the bearing capacity as a function of time after installation. The distribution of stresses in the soil volume around a

group of piles would be essential to the total behaviour of long term settlements. The work done is however an important step to understand the whole process around piles.

The work indeed considers a single pile only. The full 2D change in the stress field from pile installation (using the strain path method), however, are incorporated. The soil volume being influenced by this pile installation is up to 10 times the pile radius, i.e. less than the centre to centre distance in some pile groups. In that case we can safely say that the soil will be disturbed and the creep rates will be low. The stress state on the other hand will be much more challenging to obtain this is, as correctly pointed out, not fully investigated and is not easy to obtain at all.

5.3 Remark 3: Degree of remoulding; clay type

The degree of remoulding seems important. What are the differences between rough and smooth piles? How to achieve the best result in the view of long term bearing capacity, would extensive disturbance of the clay, by an rough surface, be an option? Would in that case steel piles, that disturb less, be an inferior choice? What are the differences between the increase in bearing capacity sensitive and non-sensitive clays?

- *As mentioned below (in the next remark) the case studied is for failure in the soil (next to the interface). We think this will be the long-term frictional failure mechanism for both steel and concrete piles (i.e. following the findings from e.g. Poulos (international) and Torstensson (Sweden) on this).*
- *With regard to differences between smooth and rough interfaces: the ‘disturbance’ is primarily from the flow of the clay around the corner of the pile base. The pile displaces the soil, pushing it downwards sideways and upwards, that is the major remoulding effect. The pile interface will not have much influence on this process. In order to minimise installation effects, larger diameter open-ended tubular piles (so they do not ‘plug’) or bored piles should be considered. They have their own downsides. For the long-term pile response this disturbance is not so bad, as the creep rate is reset by the installation (i.e. it would be higher if the soil is still intact).*
- *The discussion in the report is primarily for sensitive clays. However, and this is important to note, a sensitivity of 3–4 is already sufficient to start triggering the sensitive soil effects. In non-sensitive clays the soil will improve, a reduction in void ratio (from the installation and dissipation of excess pore water pressures) leads to higher stiffness and strength and rupture is unlikely. So the trade-off is to have either disturbed soil (with inferior stiffness and strength) and low creep rates or either less disturbance with superior stiffness and strength and perhaps larger creep rates (if the shear load is close or just passed the pre-consolidation pressure).*

5.4 Remark 4: Incorporation in design standards

In what way the advices in our existing standards (like Pålkommisionen Rapport 100, Figure 3-3) could be affected? Is the adhesion factor (α) too conservative, regarding the results? But,

B-A Torstensson found that the rupture often occurred in the clay adjacent to the pile, not in the interface of the pile/clay. Is this experience in line with your conclusions?

- *The initial α factors for ultimate bearing capacity seem well calibrated. To tentatively answer the question, it is possible to allow for higher pile bearing capacity (by means of α or β factors) as function of installation time. On the other hand, floating piles in clay are always a serviceability limit state problem, and the allowable service load is more important, so rather than 'improving' the ultimate bearing capacity, a second look at the allowable service load (and its effects on the creep rate) would be prudent.*
- *The logical next step is to incorporate load re-distribution and pile groups, but that unfortunately requires more advanced Finite Element Analyses which proves to be difficult, hence we opted for the Strain Path Method + strain driver combination. Some considerable time in this project was spend on advanced numerical analyses. The advanced non-linear constitutive models for soft soils (such as Sclay1s) do not converge well in large deformation finite element codes. This is something that requires a significant investment before reliable results are expected.*

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