

## The Analytic Hierarchy Process (AHP) for decision-making and expert judgement in railway infrastructure maintenance

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### Abstract

*Decision-making and expert judgements are two vital activities within safety and risk management, e.g. regarding: analyses of frequencies and consequences in safety and risk analysis; risk tolerability decisions in safety and risk evaluation and decision-making in safety and risk control. Hence, there are formal methodologies to support decision-making and elicit expert judgement. These methodologies also contribute to inter-subjectivity, transparency and traceability of performed decisions and judgements, which in turn support continuous improvement and risk reduction. One such methodology is the Analytic Hierarchy Process (AHP), which occurs in a steadily increasing number of scholar publications related to railway safety and risk throughout the last 30 years. This paper explores literature about AHP within railway, and describes an application of AHP to obtain preferences for strategic railway infrastructure criteria, such as safety, and for different infrastructure maintenance actions. The AHP application was supported by a software tool, which facilitated recording, calculation and presentation of the track managers' preferences. The track managers consider it easy to understand the rationale of AHP and to enter their preferences with the aid of a computer and the software tool.*

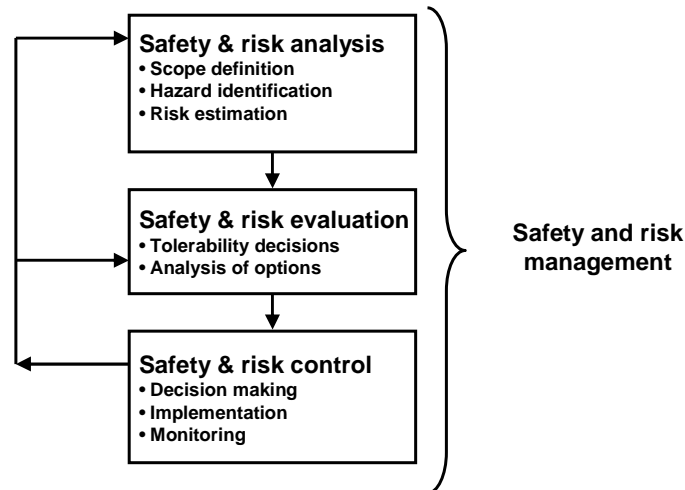
**Keywords:** Analytic Hierarchy Process (AHP), decision-making, expert judgement, railway infrastructure, maintenance, safety, risk, Sweden

### Introduction

Within the railway sector there are many requirements related to safety and risk that the stakeholders have to obey by, e.g. regarding: environment; work environment; accident prevention; security and peacetime crisis management; fire and explosive hazards; and electric power safety. To fulfil these requirements, safety and risk management are essential. Two important activities in safety and risk management are decision-making and expert judgment, e.g. regarding: analyses of frequencies and consequences for risk estimation within safety and risk analysis; tolerability decisions in safety and risk evaluation; and decision-making for safety and risk control (see Figure 1).

The safety and risk management process displayed in Figure 1 contains feedback loops, which makes it analogous to the Improvement Cycle as described by Shewhart (1939) and Deming (1994). Hence, the feedback enables continuous improvement and risk reduction, e.g. through a review of the results of performed decisions and judgements. The reason is that a decision is classified as good or bad depending on its results, which only can be done in retrospect (NUREG, 1981). However, Shewhart (1939), Deming (1994) and their followers within the quality movement also emphasise the importance of making fact-based decisions. This kind of informed decisions can be classified as correct. A correct decision is based on relevant data that is identified, collected and analysed in a rational way (NUREG, 1981). A correct decision can also be easier to change for the better if the results turn out to be unsatisfactory. Therefore, there are many supporting methodologies and tools that have been developed to enhance the ability to initially make correct decisions and judgments, especially within areas

related to safety and risk (see, e.g. Nowlan & Heap, 1978; DoD, 1980; NUREG, 1981; DoD, 1993; IEC, 1995; and Stamatis, 1995). Another benefit with these methodologies and tools is that they, when implemented and used properly, can also contribute to inter-subjectivity, transparency and traceability of performed decisions and judgements. This will in turn support continuous improvement and risk reduction, e.g. through changes of made decisions and judgments, which often is stressed as a necessity (see, e.g. Nowlan & Heap, 1978; DoD, 1980; IEC, 1995; and Stamatis, 1995).

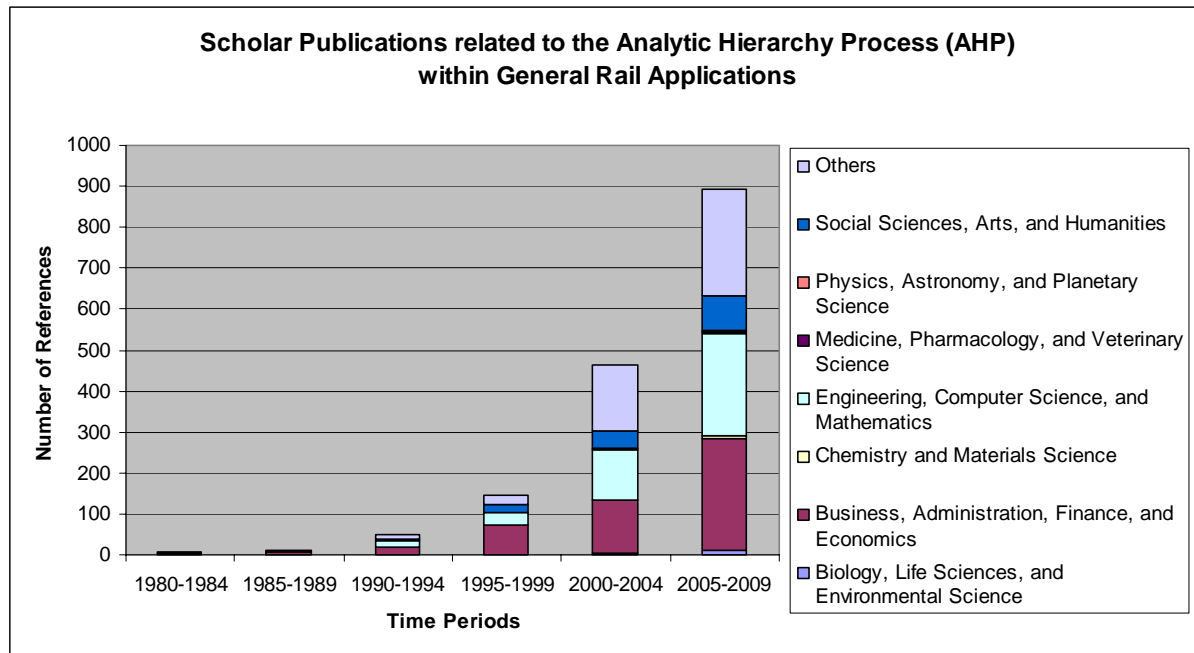


**Figure 1.** A generic safety and risk management process with phases where both decision-making and expert judgment are vital activities. Adapted from IEC (1995).

Some methodologies that support decision making is based on risk perception theory, which tries to explain the subjective judgment that people make about the characteristics and severity of a risk. Early approaches of risk perception assumed that individuals behave in a rational manner, weighing information before making a decision (Douglas, 1985). One key contribution of this early research reported the finding that people will accept risks 1,000 greater if they are voluntary than if they are involuntary, see Starr (1969). Today, three major approaches of risk perception theory can be classified as: psychology; anthropology and interdisciplinary (Wikipedia, 2009). Early research within the psychology approach tried to understand how people process information, while later work identified numerous factors responsible for influencing individual perceptions of risk, e.g. dread, newness and stigma (ibid). One key finding of this research is that the choice between different alternatives is highly affected by whether the question is stated as a loss or a gain (Tversky & Kahneman, 1981; Tversky & Thaler, 1990). The anthropology approach posits risk perceptions as produced by and supporting social institutions, where perceptions are socially constructed by institutions, cultural values, and ways of life, see e.g. Douglas & Wildavsky (1982). This means that increased social constraints limit the individual negotiation and personal control. Interdisciplinary approaches to risk perception combines research in e.g. psychology, anthropology, and communications theory to outline how communications of risk events pass from the sender and filtered through intermediate stations (e.g. individuals, groups and media) to a receiver and in the process serve to attenuate or amplify perceptions of risk (see, e.g. Kaspersen et al., 1988). Behaviours of individuals and groups then generate secondary social or economic impacts, and simultaneously increases or decreases the risk itself.

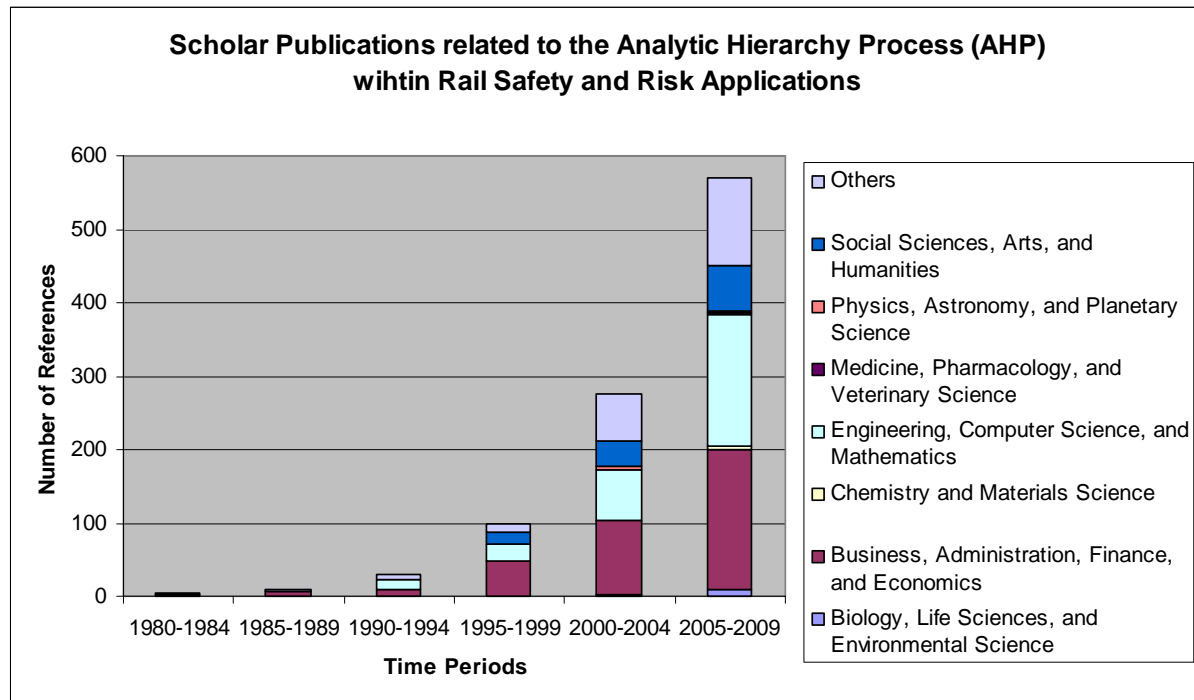
The problem of selecting the best alternatives for achieving different objectives is also studied in the field of Multi-Criteria Decision Making (MCDM); see e.g. Triantaphyllou (2000) for an

overview of different MCDM methodologies. There are also a number of formal methodologies to elicit expert judgements, e.g.: the Delphi methodology; absolute probability judgements; category ranking and paired comparison (IEC, 1995). One such methodology, which supports both decision making and elicitation of expert judgment, is the Analytic Hierarchy Process (AHP), which was presented in a book by Saaty (1980). Since then, the number of publications that contain AHP and railway have grown exponentially (see Figure 2), of which a major share is associated with safety and risk (see Figure 3).



**Figure 2.** Number of scholar publications related to the Analytic Hierarchy Process (AHP) within general rail applications, displayed for six different five year periods and eight different subject areas. Based on information retrieved through Google Scholar (30/06/2009). See Appendix A for details.

When dealing with rail safety and risk, accidents and incidents make it evident that maintenance is one critical area to consider (Itoh et al., 2004; Farrington-Darby et al., 2005; Holmgren, 2005; Wilson & Norris, 2005). There are also more than 230 scholar publications during the years 1980-2009 that refer to maintenance and AHP together with rail safety and risk. However, when further limiting the scope to infrastructure maintenance, there are only handful scholar publications that include the search words, i.e. Bookbinder & Tan (2003), Ling et al. (2006), Shetha et al. (2006), Granström (2008) and Nyström & Söderholm (2008). Furthermore, only two of these publications actually apply AHP. Hence, this paper aims to contribute to this important, but relatively unexplored area, by describing an application of AHP and its feasibility to obtain preferences for strategic railway infrastructure criteria, such as safety, and for different infrastructure maintenance actions from track managers in Sweden.



**Figure 3.** Number of scholar publications related to the Analytic Hierarchy Process (AHP) within rail safety and risk applications, displayed for six different five year periods and eight different subject areas. Based on information retrieved through Google Scholar (30/06/2009). See Appendix A for details.

### The Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) is a structured methodology for dealing with complex decisions by supporting decision makers to find the decision that best suits their needs and their understanding of the problem. AHP is based on mathematics and psychology and was developed by Thomas L. Saaty in the 1970's. Since, then AHP has been extensively studied and refined, which can be illustrated by the biennial conference on algorithms for multi-criteria decision analysis, particularly the AHP and its extension; the Analytic Network Process (ANP), see, e.g. ISAHP (2009).

When using the AHP, the decision problem is first decomposed into a hierarchy of more easily comprehended sub-problems, each of which can be analysed independently. Once the hierarchy is built, the decision makers systematically evaluate its various elements by comparing them to one another two at a time. In making the comparisons, the decision makers can use concrete data about the elements, or they can use their judgments about the elements' relative meaning and importance. In fact, one fundamental presumption of AHP is that human judgments, and not just the underlying information, can be used in performing the evaluations. (ISAHP, 2009)

However, human judgements are not entirely consistent. As an illustration, say that the decision-maker regards alternative A to be three times better than alternative B, i.e.  $A = 3B$ . The decision-maker also considers B to be twice as good as C; i.e.  $B = 2C$ . Hence, if consistent, the decision maker should consider A to be six times better than C; i.e.  $A = 6C$ . However, the decision-maker might consider C to be better than A (i.e.  $C > A$ ), or consider A to be just five times better than C (i.e.  $A = 5C$ ). In both cases, there is an inconsistency, which can be quantified by AHP. The inconsistency ratio is an indicator of the reliability of the resulting priorities. Inconsistency might be thought of as the required adjustment to improve

the consistency of the pair-wise comparisons (Saaty, 1994). AHP is also considered to be easy to use for the layman. A drawback with the AHP methodology is the time required to make the comparisons, which increases rapidly as the number of alternatives,  $n$ , increases (Saaty, 1980, 1994). See Appendix B for some further details about AHP.

### **Methodology and material**

This paper is based on two complementary studies related to the Analytic Hierarchy Process (AHP) within rail applications. One is an explorative literature study and the other is part of an extensive empirical study.

#### *Literature study*

The literature study aimed to complement the part of the empirical study that covered the usage of AHP for decisions related to infrastructure maintenance. Hence, the research question that the literature study explored was:

- What characterises the usage of the Analytical Hierarchy Process (AHP) within rail applications?

To collect data, the search engine Google Scholar was judged to be sufficient as a supporting tool. The reason is that Google Scholar provides one access point to peer-reviewed abstracts, books, papers, theses and other scholarly literature from many broad areas of research. It is further possible to find works from a wide variety of academic publishers and professional societies, as well as scholarly articles available across the web. See Google (2009) for further information about the search engine.

The search was performed during the time period 30/06/2009-01/07/2009. Based on the intentions of the empirical study, applied search words in different combinations were: “Rail”; “Analytic Hierarchy Process”; “Safety”; and “Risk”. The search was aimed at articles that contained the words anywhere in the text, in order to retrieve as many related publications as possible. The search was also divided into six different five year periods between 1980 and 2009 to enable an examination of any changes in the publication of AHP within rail applications over the years. The selection of start date was based on Saaty’s original publication of a book about AHP in 1980 and the end date was the day of the search. Furthermore, the search was performed individually for seven different subject areas provided by the search engine, to enable an examination of their respective share of rail applications. To narrow the search further, the search words “maintenance” and “infrastructure maintenance” were added in two consecutive steps. Finally, Microsoft Excel was applied as a supporting tool in the data analysis to make calculations and create Tables (see Tables A1-A6) and Figures (see Figures 2 and 3).

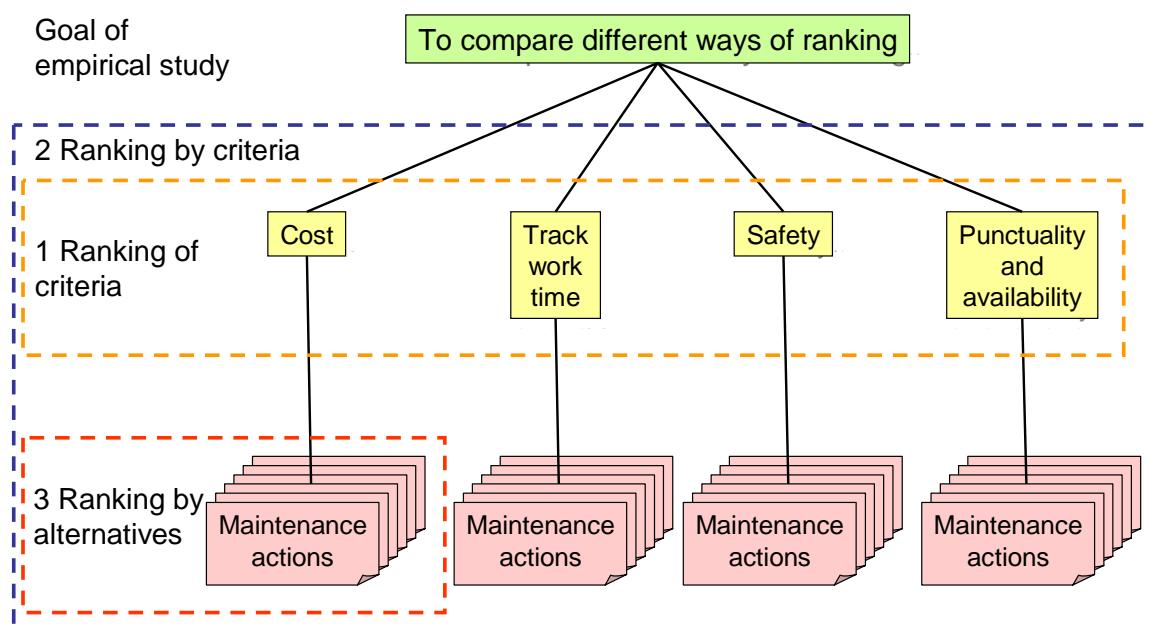
#### *Empirical study*

The empirical material presented in this paper is based on a study originally presented in Nyström & Söderholm (2008), which in turn is part of a larger research effort presented in Nyström (2008). The research questions related to the material presented in this paper were:

- How important do decision-makers consider different criteria affected by infrastructure maintenance to be?
- How consequent is the selection of maintenance actions?

AHP was applied to answer the first of these research questions, by obtaining the ranking of eight criteria (e.g. safety) based on a pair-wise comparison of all criteria. AHP was also used to answer the second of these research questions, by obtaining two different ways of ranking the same alternative maintenance actions. One ranking was based on a pair-wise comparison of all (eight or 12) maintenance alternatives with regard to one criteria at a time. The other ranking was based on pair-wise comparison of all maintenance alternatives (eight or 12) without any other formal consideration. Then, the results from the two different ways of ranking were compared with each other, which gave a measure of the consistency in selection of maintenance alternatives. Hence, the AHP was used in three different ways (see also Figure 4):

1. Each one of the eight criteria (effects of maintenance) was pair-wise compared to all the others. This procedure gave the ‘criteria prioritisation’.
2. Each maintenance action was compared pair-wise to every other maintenance action, with respect to each individual criterion. The result of this procedure is called a ‘ranking by criteria’.
3. Each maintenance action was compared pair-wise to every other maintenance action, without using any criterion. The result of this procedure is called a ‘ranking by alternatives’.



**Figure 4.** A simplified example of the applied hierarchy with goal, criteria and alternatives. The goal is green, the criteria are yellow, and the alternatives are pink. The diagram has been simplified by showing only four out of the eight criteria used in the empirical study and the alternatives as stacks of papers, eliminating the large number of lines that actually connect them to their covering criteria.

The Expert Choice® software was used as a supporting tool for elicitation, recording and analysis of the decision-makers preferences through AHP experiments. Some complementary analyses were performed with the aid of the software tool Microsoft Excel.

The track manager was identified as a key role in decisions regarding infrastructure maintenance. Hence, track managers were involved in the establishment of criteria through a

group interview, as subjects in the AHP experiments and in individual interviews to obtain other relevant information.

Besides the group interview with track managers, measures in the strategic plan of Banverket (2006) were used to establish eight strategic criteria and get a reasonable correspondence to the four perspectives of Kaplan and Norton's (1996) balanced scorecard, i.e. 'Financial', 'Internal', 'Learning' and 'Customer'.

## Results

The performed literature study indicates that the use of the Analytic Hierarchy Process (AHP) within both general rail applications and specific rail safety and risk applications has grown exponentially the last three decades. See Appendix A and Figures 2 and 3.

Rail safety and risk applications make out the majority of all rail applications that use AHP for the last three decades. The share of these applications varies between 60 and 80 percent for each of the six different five year periods. There is no obvious trend in the change of this share. See Appendix A and Figures 2 and 3. Within these publications, there are more than 230 publications related to maintenance. However, when considering infrastructure maintenance, there are only handful scholar publications.

When considering the use of AHP, the number of references with rail risk applications is larger for each of the six studied time periods than the number of references with rail safety applications.

The three subject areas that make out the majority of railway AHP-applications are, in decreasing order: 1) 'business, administration, finance, and economics'; 2) 'engineering, computer science, and mathematics'; and 3) 'social sciences, arts, and humanities'. This order is valid for both general rail applications and for specific rail safety and risk applications. See Appendix A and Figures 2 and 3.

The criteria that were developed in the empirical study are shown in Table 4. Each criterion has a maximum of three subsequent factors, and both the criteria and their factors are defined.

In Table 5, the rankings of the eight criteria according to each of the six track manager (S1-S6) are shown. In the two right-most columns are the mean of the priorities of the different track managers, and, from these weights, the resulting overall ranking.

As seen in Table 5, five out of six track managers ranked the 'Safety' criterion as most important out of the eight included criteria. The track manager with a deviating view regarded 'Safety' as a hygiene factor, assuming that it is already in place, and therefore ranked it as the second most important criterion. One track manager ranked the criterion 'Environmental impact' as second most important, while the other five track managers ranked it as sixth or eighth. As can be seen from Table 5, 'Safety' is the top-ranked criterion, accounting for 40.6% of the total priority. 'Punctuality and availability' is second, while 'Track work time', 'Cost' and 'Condition' constitute the middle section. The lowest priorities, around 6%, were given to 'Own abilities and development', 'Collaboration with stakeholders' and 'Environmental impact'.

**Table 4.** *The developed criteria, as they were presented to the track managers in the AHP experiments. Each criterion has a definition and a number of factors in order to facilitate the track manager's understanding of what is included in the criteria. Here, the definition and factors are shown only for criterion 3 'Safety'.*

| <b>Criterion</b>  |
|---|
| 1. Cost   |
| 2. Track work time  |
| 3. Safety<br>Definition of Safety: the absence of accidents and incidents, including level crossing accidents and suicides. This includes personnel at Banverket, other personnel within the railway sector and the general public. |
| 3.1 Number of deaths and seriously injured  |
| 3.2 Number of accidents and incidents   |
| 4. Punctuality and availability   |
| 5. Condition  |
| 6. Environmental impact   |
| 7. Own abilities and development  |
| 8. Collaboration with stakeholders  |

**Table 5.** *The rankings of the criteria of the six different track managers. The priority column shows the arithmetic mean of the priorities of the six track managers for the respective criterion (not shown), which gives the criterion's rank.*

| Criterion                       | Track manager |    |    |    |    |    | Priority | Rank |
|---------------------------------|---------------|----|----|----|----|----|----------|------|
|                                 | S1            | S2 | S3 | S4 | S5 | S6 |          |      |
| Safety                          | 1             | 1  | 2  | 1  | 1  | 1  | 0.406    | 1    |
| Punctuality and availability    | 2             | 3  | 1  | 3  | 5  | 2  | 0.154    | 2    |
| Track work time                 | 3             | 2  | 3  | 5  | 4  | 5  | 0.093    | 3    |
| Cost                            | 4             | 4  | 4  | 7  | 7  | 3  | 0.085    | 4    |
| Condition                       | 5             | 8  | 5  | 4  | 2  | 4  | 0.084    | 5    |
| Own abilities and development   | 8             | 7  | 7  | 8  | 3  | 7  | 0.063    | 6    |
| Collaboration with stakeholders | 7             | 5  | 6  | 6  | 6  | 8  | 0.058    | 7    |
| Environmental impact            | 6             | 6  | 8  | 2  | 8  | 6  | 0.057    | 8    |

In Table 6, the inconsistency ratio of the criteria prioritisation for the different track managers is shown. One possible explanation for the high inconsistencies of two track managers (S2 and S3) is that the subjects were not allowed to reconsider the pair-wise comparisons that contributed the most to the inconsistency. However, such changes were allowed for the pair-wise comparisons of the maintenance actions later in the experiment.

**Table 6.** *The inconsistency of the criteria prioritisation for each of the track managers (S1-S6).*

| Track manager                                   | S1   | S2   | S3   | S4   | S5   | S6   |
|---|------|------|------|------|------|------|
| <b>Inconsistency of criteria prioritisation</b> | 0.01 | 0.24 | 0.28 | 0.03 | 0.11 | 0.11 |

Table 7 shows the inconsistencies of the pair-wise comparisons of the alternative maintenance actions, with respect to each criterion (1-8) for each individual track manager (S1-S6). The



two highest inconsistencies of each track manager are starred (if 0.10 or greater). The last row shows the inconsistencies of the pair-wise comparisons with respect to the alternative maintenance actions. The exceptionally low inconsistency of S3 is at least partially explained by the fact that he entered mostly small and moderate preferences. Three out of the six track managers had ‘Track work time’ as the criterion with the highest inconsistency. This indicates that the definition of the criterion is unclear or that the track work time is unknown or hard to assess.

**Table 7.** *The inconsistencies of the comparisons carried out by the track managers (S1-S6), with respect to each of the eight criteria and with respect to the maintenance alternatives. The two highest inconsistencies of each track manager are starred (if 0.10 or greater).*

| Criterion                          | S1    | S2    | S3   | S4    | S5    | S6    |
|------------------------------------|-------|-------|------|-------|-------|-------|
| 1. Cost                            | 0.11  | 0.34* | 0.01 | 0.08  | 0.11  | 0.05  |
| 2. Track work time                 | 0.26* | 0.02  | 0.00 | 0.41* | 0.19* | 0.13* |
| 3. Safety                          | 0.22  | 0.16* | 0.00 | 0.05  | 0.08  | 0.05  |
| 4. Punctuality and availability    | 0.71* | 0.14  | 0.00 | 0.01  | 0.15* | 0.05  |
| 5. Condition                       | 0.08  | 0.04  | 0.00 | 0.04  | 0.04  | 0.01  |
| 6. Environmental impact            | 0.01  | 0.07  | 0.01 | 0.01  | 0.06  | 0.03  |
| 7. Own abilities and development   | 0.02  | 0.15  | 0.00 | 0.01  | 0.01  | 0.01  |
| 8. Collaboration with stakeholders | 0.00  | 0.08  | 0.00 | 0.01  | 0.01  | 0.02  |
| Maintenance alternatives           | 0.00  | 0.12  | 0.00 | 0.01  | 0.09  | 0.01  |

The degree of correlation between two rankings (i.e. only the order is taken into account, not the priorities from which it was calculated) might be calculated using the Spearman rank correlation coefficient,  $r_s$  (Sachs, 1982). The coefficient  $r_s$  attains values between and including -1 and 1, with 1 meaning that the two rankings are identical and -1 meaning that they are reversed. Table 8 shows  $r_s$  for the ranking by criteria and ranking by alternatives, for each of the track managers (S1-S6). The rankings of S6 have the highest correlation ( $r_s = 0.85$ ). For the rankings with the lowest absolute correlation, the rankings of S3 ( $r_s = 0.41$ ), the hypothesis that the rankings are independent is rejected at the 20 percent level. For the other track managers it is rejected at the 10 percent or lower levels. Track manager S4 has a negative correlation coefficient, which means that the rankings are, to some extent, the reverse of each other.

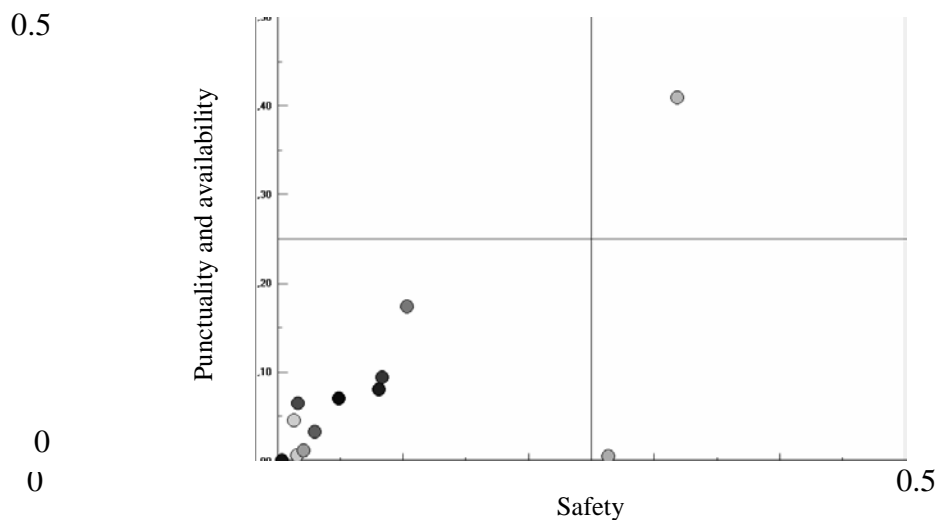
**Table 8.** *The Spearman rank correlation coefficient,  $r_s$ , between the ranking by criteria and the ranking by alternatives, for the track managers (S1-S6).*

| Track manager | S1   | S2   | S3   | S4    | S5   | S6   |
|---------------|------|------|------|-------|------|------|
| $r_s$         | 0.69 | 0.69 | 0.41 | -0.54 | 0.60 | 0.85 |

If the priorities of the criteria for the alternative maintenance actions are correlated, it might imply different things. A correlation might imply that these criteria do co-vary with the maintenance actions. It might also mean that correlated criteria are not formulated in a mutually exclusive way or reflect a transfer from one criterion to another. This is motivated by the fact that the track managers participating in the experiment might have experienced some fatigue towards the end of the experiment, which might have caused them to consider the previous instead of the current criterion.

According to three of the six track managers, criterion ‘3 Safety’ is strongly correlated with both criterion ‘4 Punctuality and availability’ and criterion ‘5 Condition’. This correlation

might be explained by the assumption that improved condition leads to increased reliability, which in turn results in improved safety as well as increased availability and thus punctuality. An example, representative of these correlations, is given in Figure 5, where the correlation between '3 Safety' and '4 Punctuality and availability' for track manager S1 is illustrated. For example, the maintenance action 'to exchange rail that has been classified as unsafe', has the highest priority regarding both '3 Safety' and '4 Punctuality and availability'. Figure 5 also contains an outlier, 'Installation of protection wall', for which the priority with respect to 'Safety' is about 0.27, but the priority with respect to 'Punctuality and availability' is close to 0.00.



**Figure 5.** The priorities of criterion '3 Safety' are on the x-axis, the priorities of criterion '4 Punctuality and availability' are on the y-axis. The twelve maintenance alternatives given to track manager S1 are plotted.

The judgements of two of the six track managers (S1 and S6) show a correlation between criterion '3 Safety' and criterion '2 Track work time' for their respective maintenance actions. Maintenance actions that are judged to contribute to high safety also tend to be judged to require long track work time. Examples of maintenance actions that show this correlation include frost insulation of the track bed, which is judged to have strong impact on increased safety and also to require long track work time, according to S1. Another example is real estate measures, which track manager S6 judges to impact safety to a low degree and to require short track work time, while turnout adjustment is among the maintenance actions that S6 judges to have high impact on safety and require long track work time. However, a correlation between safety and track work time is not seen for the other four track managers.

## Discussion

Based on the literature study, it is not unreasonable to expect a future exponential increase in the use of AHP within general rail applications, as well as within specific rail safety and risk applications. However, it should be noted that the applied search engine has its limitations, e.g. the way that the results are obtained and the classification of results into subject areas are not always transparent.

There is no indication that the top three subject areas for AHP use in rail applications will change in a near future. An emerging subject area might be that of 'biology, life sciences, and environmental science'. One typical example of this is Tuzkaya (2009), who uses a combination of fuzzy AHP and preference ranking organisation methodology for choice of

transportation modes with regard to their environmental effects. However, there are yet too few publications encountered in the performed literature search to draw any conclusion about this development. On the other hand, the growing focus on sustainability and sustainable development will probably contribute to such a development. This change might also be reflected in the track managers' prioritisation of strategic criteria, where one ranked the criterion 'Environmental impact' as second most important, while the other five track managers ranked it as sixth or eighth.

In the empirical study presented in this paper, the decisions of track managers have been investigated with the aid of AHP. By comparing two different ways of ranking; by criteria and by alternatives, possible variations in the decisions are indicated. However, a low variation indicates only that correct decisions are made, not whether they are good or bad.

One finding of the empirical study was that non-documented actions are a problem. Documentation as to the grounds for a certain decision and comparisons of the results obtained to the ones planned are also lacking. Therefore, the scope for evaluating decisions made and identifying whether they were correct or good, and thereby achieve continuous improvement and risk reduction, is undermined. The need for documentation is stressed by the long life of some railway infrastructure items; longer than a human's working life. Documentation would also enable continuous improvement and risk reduction, based on an evaluation of decisions made. Hence, it is proposed that preferences for made decisions are recorded as they are in this paper, in order to document the rationale of the decisions and to facilitate mutual learning among decision-makers and over time. One reason for this is that today most of the track managers use a list of desired actions that provides too little description to make informed selections, i.e. correct decisions.

The AHP methodology is judged to be applicable to selection among different maintenance actions related to railway infrastructure. The track managers easily accepted the rationale of the methodology and found it on the whole easy to work with the software. The major drawback with the AHP methodology was the long experiment time, 4–5 hours, needed to do all pair-wise comparisons.

The track managers roughly agree on the prioritisation of criteria. However, the discrepancies between the results of the two ways employed to elicit the preferences for the maintenance actions are rather large. The lowest priorities were given to the three criteria 'Own abilities and development', 'Collaboration with stakeholders' and 'Environmental impact'. It might be interesting to notice that these criteria were developed by a study of Banverket's strategic plan to achieve a closer correspondence to the logic of the Balanced Scorecard than was obtained by just using criteria elicited by interviewing track managers. Since some track managers were used in the interviews and others in the AHP experiments, this finding would indicate that the track managers in the study have a rather mutual view of which criteria to prioritise (which to some degree differs from the strategic plan of Banverket).

Considering the 'Safety' criterion, a strong positive correlation to the criteria 'Punctuality and availability' and 'Condition' was found for three out of the six track managers when ranking the maintenance actions. This can probably be explained by the intension of maintenance actions to retain or restore the required function of the railway infrastructure, which in turn is expected to have a relationship between a good condition (and reliability performance) of the railway infrastructure and improved safety, as well as improved availability and thereby punctuality. However, depending on the specific maintenance action, there might also be a

correlation between the achieved safety level and the required track work time. Within this study, there is a correlation between high safety and long track work time for the included maintenance actions.

#### *Further research*

This paper has investigated the track manager as the sole decision-maker. However, the information and knowledge that a track manager gets from other professionals is embedded in their selections. The AHP might also be used for simultaneously considering the preferences of multiple decision-makers, e.g. infrastructure managers and centralised train traffic control centres.

This paper has considered the selection among maintenance alternatives that lead to different types of effects. Another application of the AHP is to choose among different alternatives that strive for the same aim, e.g. increased level crossing safety or reduced risk of derailment.

Another possibility for further study is to apply the Analytic Network Process (ANP), which is a generalisation of AHP. One important feature with ANP, compared with AHP, is that it also allows interaction and feedback. See, e.g. ISAHP (2009).

In this study the prioritisation of maintenance actions with regard to the criteria 'Cost' and 'Environmental impact' has been based on their positive impact on the criteria. Hence, a maintenance action resulting in low cost or environmental impact has received a higher prioritisation than a maintenance action that increases these criteria. This approach is the traditional way of using AHP. However, one alternative approach would be to calculate the quotient between 'good' criteria (e.g. 'Condition' and 'Punctuality and availability') and 'bad' criteria (i.e. 'Cost' and 'Environmental impact') and select the alternatives that receive the greatest marginal. Another alternative approach would be to apply negative numbers for the prioritisation of maintenance actions in relation to the 'bad' criteria and combine them in some way with the other 'positive' criteria. One reason to use negative numbers in the prioritisation within AHP is that it is claimed to give more relevant results than the traditional application of AHP. See e.g. Saaty & Ozdemir (2003).

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**Appendix A: Some Results from the Literature Study**

**Table A1.** Summary of search results for different subject areas when using the combination: “Rail” AND “Analytic Hierarchy Process”. This is assumed to indicate the overall application of the Analytic Hierarchy Process (AHP) within the railway sector. See Figure 2 for a depiction of the table.

| Subject Area                                      | Rail AND "Analytic Hierarchy Process" |           |           |            |            |            |
|---|---------------------------------------|-----------|-----------|------------|------------|------------|
|   | 1980-1984                             | 1985-1989 | 1990-1994 | 1995-1999  | 2000-2004  | 2005-2009  |
| Biology, Life Sciences, and Environmental Science | 0                                     | 0         | 0         | 0          | 3          | 13         |
| Business, Administration, Finance, and Economics  | 3                                     | 8         | 18        | 71         | 131        | 271        |
| Chemistry and Materials Science                   | 0                                     | 0         | 0         | 0          | 0          | 8          |
| Engineering, Computer Science, and Mathematics    | 0                                     | 0         | 17        | 31         | 122        | 249        |
| Medicine, Pharmacology, and Veterinary Science    | 0                                     | 0         | 0         | 0          | 0          | 4          |
| Physics, Astronomy, and Planetary Science         | 0                                     | 0         | 0         | 0          | 5          | 1          |
| Social Sciences, Arts, and Humanities             | 3                                     | 1         | 3         | 19         | 42         | 86         |
| Others  | 0                                     | 2         | 10        | 23         | 160        | 260        |
| <b>TOTAL</b>                                      | <b>6</b>                              | <b>11</b> | <b>48</b> | <b>144</b> | <b>463</b> | <b>892</b> |

**Table A2.** Summary of search results for different subject areas when using the combination: “Rail” AND “Analytic Hierarchy Process AND/OR “Safety” AND/OR “Risk”. This is assumed to indicate the specific application of the Analytic Hierarchy Process (AHP) for safety and risk purposes within the railway sector. See Figure 3 for a depiction of the table.

| Subject Area                                      | Rail AND "Analytic Hierarchy Process" AND/OR Safety AND/OR Risk |           |           |           |            |            |
|---|---|-----------|-----------|-----------|------------|------------|
|   | 1980-1984   | 1985-1989 | 1990-1994 | 1995-1999 | 2000-2004  | 2005-2009  |
| Biology, Life Sciences, and Environmental Science | 0   | 0         | 0         | 0         | 3          | 10         |
| Business, Administration, Finance, and Economics  | 2   | 7         | 10        | 49        | 101        | 189        |
| Chemistry and Materials Science                   | 0   | 0         | 0         | 0         | 0          | 6          |
| Engineering, Computer Science, and Mathematics    | 0   | 0         | 12        | 23        | 69         | 178        |
| Medicine, Pharmacology, and Veterinary Science    | 0   | 0         | 0         | 0         | 0          | 4          |
| Physics, Astronomy, and Planetary Science         | 0   | 0         | 0         | 0         | 5          | 1          |
| Social Sciences, Arts, and Humanities             | 2   | 1         | 2         | 15        | 33         | 62         |
| Others  | 0   | 1         | 7         | 12        | 64         | 119        |
| <b>TOTAL</b>                                      | <b>4</b>  | <b>9</b>  | <b>31</b> | <b>99</b> | <b>275</b> | <b>569</b> |

**Table A3.** Summary of search results for different subject areas when using the combination: “Rail” AND "Analytic Hierarchy Process" AND “Safety” NOT “Risk”. This is assumed to indicate the specific application of the Analytic Hierarchy Process (AHP) for safety purposes within the railway sector.

| Subject Area                                      | Rail AND "Analytic Hierarchy Process" AND Safety NOT Risk |           |           |           |           |            |
|---|---|-----------|-----------|-----------|-----------|------------|
|   | 1980-1984   | 1985-1989 | 1990-1994 | 1995-1999 | 2000-2004 | 2005-2009  |
| Biology, Life Sciences, and Environmental Science | 0   | 0         | 0         | 0         | 0         | 0          |
| Business, Administration, Finance, and Economics  | 0   | 2         | 1         | 10        | 17        | 31         |
| Chemistry and Materials Science                   | 0   | 0         | 0         | 0         | 0         | 2          |
| Engineering, Computer Science, and Mathematics    | 0   | 0         | 1         | 5         | 14        | 32         |
| Medicine, Pharmacology, and Veterinary Science    | 0   | 0         | 0         | 0         | 0         | 1          |
| Physics, Astronomy, and Planetary Science         | 0   | 0         | 0         | 0         | 2         | 0          |
| Social Sciences, Arts, and Humanities             | 0   | 0         | 0         | 2         | 6         | 8          |
| Others  | 0   | 1         | 2         | 3         | 23        | 43         |
| <b>TOTAL</b>                                      | <b>0</b>  | <b>3</b>  | <b>4</b>  | <b>20</b> | <b>62</b> | <b>117</b> |

*Appendix A: Some Results from the Literature Study*

**Table A4.** Summary of search results for different subject areas when using the combination: “Rail” AND “Analytic Hierarchy Process” AND “Risk” NOT Safety”. This is assumed to indicate the specific application of the Analytic Hierarchy Process (AHP) for risk purposes within the railway sector.

| Subject Area                                      | Rail AND "Analytic Hierarchy Process" AND Risk NOT Safety |           |           |           |            |            |
|---|---|-----------|-----------|-----------|------------|------------|
|   | 1980-1984   | 1985-1989 | 1990-1994 | 1995-1999 | 2000-2004  | 2005-2009  |
| Biology, Life Sciences, and Environmental Science | 0   | 0         | 0         | 0         | 2          | 7          |
| Business, Administration, Finance, and Economics  | 2   | 3         | 2         | 22        | 39         | 80         |
| Chemistry and Materials Science                   | 0   | 0         | 0         | 0         | 0          | 3          |
| Engineering, Computer Science, and Mathematics    | 0   | 0         | 6         | 6         | 26         | 70         |
| Medicine, Pharmacology, and Veterinary Science    | 0   | 0         | 0         | 0         | 0          | 1          |
| Physics, Astronomy, and Planetary Science         | 0   | 0         | 0         | 0         | 1          | 0          |
| Social Sciences, Arts, and Humanities             | 1   | 1         | 1         | 7         | 11         | 25         |
| <i>Others</i>                                     | 0   | 0         | 1         | 4         | 22         | 49         |
| <b>TOTAL</b>                                      | <b>3</b>  | <b>4</b>  | <b>10</b> | <b>39</b> | <b>101</b> | <b>235</b> |

**Table A5.** The percentage of rail safety and risk applications out of all rail applications that uses the analytical Hierarchy Process (AHP).

| Subject Area   |           |           |           |           |           |           |
|--|-----------|-----------|-----------|-----------|-----------|-----------|
|  | 1980-1984 | 1985-1989 | 1990-1994 | 1995-1999 | 2000-2004 | 2005-2009 |
| Number of general rail applications that uses AHP  | 6         | 11        | 48        | 144       | 463       | 892       |
| Number of rail safety and risk applications that uses AHP                                    | 4         | 9         | 31        | 99        | 275       | 569       |
| Percentage of rail safety and risk applications (out of all rail applications) that uses AHP | 66,7      | 81,8      | 64,6      | 68,8      | 59,4      | 63,8      |

**Table A6.** Scholar publications that contain different combinations of the words “Rail”, “Safety”, “Risk”, “Infrastructure Maintenance” and the “Analytic Hierarchy Process” (AHP) in the time period of 1980-2009.

| Subject Area                                     | Rail AND "Analytic Hierarchy Process" AND "Infrastructure Maintenance" AND 'Different Combinations of Safety/Risk' |           |           |           |                   |   |
|--|--|-----------|-----------|-----------|-------------------|---|
|  | 1980-1984  | 1985-1989 | 1990-1994 | 1995-1999 | 2000-2004         | 2005-2009                                 |
| Business, Administration, Finance, and Economics | 0  | 0         | 0         | 0         | 1<br>(NOT Safety) | 1<br>(NOT Risk)<br>1<br>(Safety AND Risk) |
| Engineering, Computer Science, and Mathematics   | 0  | 0         | 0         | 0         |                   | 2<br>(Safety AND Risk)                    |
| <b>TOTAL</b>                                     | <b>0</b>   | <b>0</b>  | <b>0</b>  | <b>0</b>  | <b>1</b>          | <b>4</b>                                  |

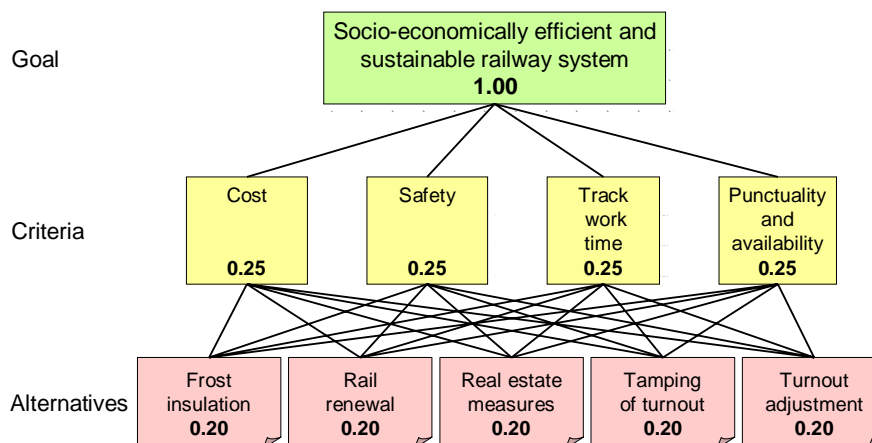
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## Appendix B: The Analytic Hierarchy Process (AHP)

The AHP process can be summarised as:

1. Model the problem as a hierarchy containing the decision goal, the alternatives for reaching it, and the criteria for evaluating the alternatives. The hierarchy can be visualised as a diagram, with the goal at the top, the alternatives at the bottom, and the criteria in the middle. See Figure B1 for an example of a hierarchy diagram. For further information about how to create a hierarchy, see e.g. Saaty (1980), Saaty (1984) and Saaty & Shih (2009).
2. Establish priorities among the elements of the hierarchy by making a series of judgments based on pair-wise comparisons of the elements. In order to sort  $n$  alternatives, one needs  $n-1$  comparisons, when the alternatives are compared pair-wise both regarding which is preferred and the degree of preference. In contrast, the AHP requires each single alternative to be compared to every other alternative, necessitating  $n(n-1)/2$  comparisons. The 'extra' comparisons introduce redundancy, which makes the resulting priorities more trustworthy. One way to perform the pair-wise comparison is to use a questionnaire with the layout displayed in Figure B2. The priorities represent the relative weights of the nodes in any group (goal, criteria or alternatives) of an AHP hierarchy. The priority of the nodes in all groups always adds up to 1.0, i.e. the goal is 1.0 and the priorities of the criteria and alternatives add up to 1.0 respectively. Priorities are, like probabilities, dimensionless and absolute numbers between zero and one. The weight can refer to, e.g. importance, preference, or likelihood, depending on the factor that is considered. The fundamental scale for comparison within different groups that is used within AHP can be found in Table B1.
3. Synthesise these judgments to yield a set of overall priorities for the hierarchy. The ranking of priorities is achieved by calculating the comparison matrix's normalised Eigenvector. In Figure B1, the nodes' associated priorities are default, summarising to 1.0 within each group of node.
4. Check the consistency of the judgments by using the Random Consistency Index (RI), as exemplified in Table B3, and calculating the Consistency Index (CI) and the Consistency Ratio (CR).
5. Come to a final decision based on the results of this process.



**Figure B1.** A simplified example of a hierarchy with goal, criteria and alternatives within the railway sector. The goal is green, the criteria are yellow, and the alternatives are pink. The numbers within each box (node) are associated default priorities (adding to 1.0 within each group of node types, i.e. goal, criteria and alternatives) before any performed judgements.

*Appendix B: The Analytic Hierarchy Process (AHP)*

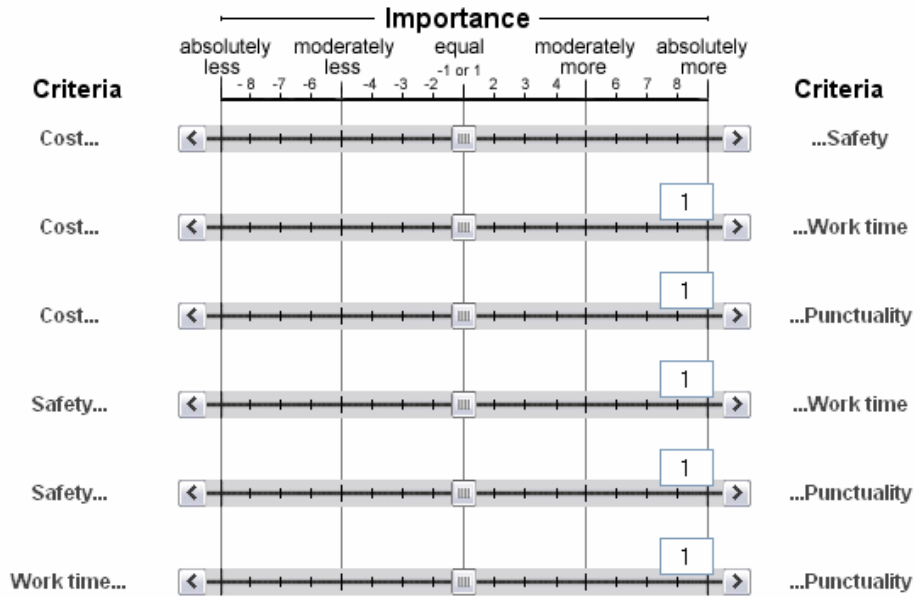
If the hierarchy in Figure B1 would be used, there would be six pair-wise comparisons of the criteria that would be entered in the upper triangular part of a four by four matrix on the criteria level (see the matrix in Table B2). Furthermore, there are five alternatives that should be pair-wise compared to each other in relation to each individual criterion. Hence, on the alternatives level there are four five by five matrices with 10 pair-wise comparisons each. In total, the hierarchy in Figure B1 would require 46 pair-wise comparisons.

**Table B1.** *The fundamental scale for pair-wise comparison in the Analytic Hierarchy Process (AHP). Adapted from Saaty (1980).*

| <b>Intensity of importance</b> | <b>Definition</b>  | <b>Explanation</b>   |
|--------------------------------|--|--|
| 1                              | Equal importance   | Two activities contribute equally to the objective   |
| 3                              | Weak importance of one over another  | Experience and judgement slightly favour one activity over another                               |
| 5                              | Essential or strong importance   | Experience and judgement strongly favour one activity over another                               |
| 7                              | Very strong or demonstrated importance   | An activity is favoured very strong over another; its dominance demonstrated in practice         |
| 9                              | Absolute importance  | The evidence favouring one activity over another is of the highest possible order of affirmation |
| 2, 4, 6, 8                     | Intermediate values between adjacent scale values  | When compromise is needed  |
| Reciprocals of above nonzero   | If activity <i>i</i> has one of the above nonzero numbers assigned to it when compared with activity <i>j</i> , then <i>j</i> has the reciprocal value when compared with <i>i</i> | A reasonable assumption  |

Eliciting judgements about the criteria in Figure B1 can be done by using a questionnaire as the one exemplified in Figure B2. The first pair of criteria in Figure B2 is cost and safety. If safety is favoured over cost the selection of a number should be to the right of the middle of the scale. The number depends of how much safety is favoured before safety, i.e. somewhere between 1 (equal importance) and 9 (absolute more important).

Appendix B: The Analytic Hierarchy Process (AHP)



**Figure B2.** A possible approach to obtain judgements by the use of questionnaire. Here four of the criteria of the empirical study of this paper are applied (as also illustrated in Figure B1). Adapted from Saaty (1980) by the use of CCI (2009).

Once all par wise comparisons are made (e.g. by using the scale of Table B1 and a questionnaire similar to the one in Figure B2), it is possible to make a matrix as the one displayed in Table B2. With four criteria, there will be six comparisons, which can be fitted within the upper half of a four by four matrix. The diagonal elements of the matrix are always 1.0, since a criterion has equal importance when compared with it self. The upper triangular matrix is initially filled up by using the following two rules:

1. If the judgment value is on the left hand side of 1.0 (see Figure B2), insert the actual judgment value in the matrix.
2. If the judgment value is on the right hand side of 1.0 (see Figure B2), insert the reciprocal value in the matrix.

To fill the lower triangular matrix, the reciprocal values of the upper diagonal are used. If  $a_{ij}$  is the element of row  $i$  and column  $j$  of the matrix, then the lower diagonal is filled using the following formula:  $a_{ji} = 1/a_{ij}$ . Hence, a complete comparison matrix is established where all values are positive, i.e.  $a_{ij} > 0$ . See Table B2 for an example of a comparison matrix.

**Table B2.** Matrix for pair-wise comparison of four criteria. The criteria given in the table is among the ones used in the empirical study presented in this paper. Note that each criteria receives the number 1.0 (equal importance) when compared with it self, i.e. the diagonal of the matrix. The values below the diagonal are the reciprocal values of the upper triangular matrix.

| Criteria                   | Cost                  | Safety time           | Track work            | Punctuality & availability |
|----------------------------|-----------------------|-----------------------|-----------------------|----------------------------|
| Cost                       | <b>1.0</b>            | $a_{12}$              | $a_{13}$              | $a_{14}$                   |
| Safety                     | $a_{21}$              | <b>1.0</b>            | $a_{23}$              | $a_{24}$                   |
| Track work time            | $a_{31} = 1 / a_{13}$ | $a_{32} = 1 / a_{23}$ | <b>1.0</b>            | $a_{34}$                   |
| Punctuality & availability | $a_{41} = 1 / a_{14}$ | $a_{42} = 1 / a_{24}$ | $a_{43} = 1 / a_{34}$ | <b>1.0</b>                 |

*Appendix B: The Analytic Hierarchy Process (AHP)*

Having a pair wise comparison matrix, a ranking of priorities is achieved by calculating the matrix's normalised Eigenvector. A short computational way to obtain this ranking is to:

1. raise the pair-wise matrix to powers that are successively squared each time.
2. calculate and normalise the row sums, which gives the Eigenvector.
3. stop the calculations once the difference between the sums in two consecutive calculations are smaller than a prescribed value.

For a consistent reciprocal matrix, the largest Eigenvalue is equal to the size of the comparison matrix, i.e.  $\lambda_{\max} = n$ . The Consistency Index (CI) is a measure of the deviation, or degree of consistency, and is calculated as:  $CI = (\lambda_{\max} - n) / (n - 1)$ . The CI can then be compared with the appropriate consistency index, called the Random Consistency Index (RI), see Table B3 for examples of RI.

**Table B3.** *Random Consistency Index (RI). Adapted from Saaty (1994).*

|           |   |   |      |      |      |      |      |      |      |      |
|-----------|---|---|------|------|------|------|------|------|------|------|
| <i>n</i>  | 1 | 2 | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
| <i>RI</i> | 0 | 0 | 0.52 | 0.89 | 1.11 | 1.25 | 1.35 | 1.40 | 1.45 | 1.49 |

A comparison of CI and RI is done by the calculation of the Consistency Ratio (CR) according to the formula:  $CR = CI / RI$ . Then, CR is used to decide if the judgments are consistent or not. A CR that is 10% or less is considered as an indication of consistency in performed judgements. However, if CR is greater than 10%, the judgements have to be revised.

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