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Proportional Intensity Model considering Imperfect Repair for Repairable Systems

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Abstract: The Proportional Intensity Model (PIM) extends the classical Proportional Hazard Model (PHM) in order to deal with repairable systems. This paper develops a more general PIM model which uses the imperfect model as baseline function. By using the imperfect model, the effectiveness of repair has been taken into account, without assuming an "as-bad-as-old" or an "as-good-as-new" scheme. Moreover, the effectiveness of other factors, such as the environmental conditions and the repair history, is considered as covariant in this PIM. In order to solve the large number parameters estimation problem, a Bayesian inference method is proposed. The Markov Chain Monte Carlo (MCMC) method is used to compute the posterior distribution for the Bayesian method. The Bayesian Information Criterion (BIC) is employed to perform model selection, namely, selecting the baseline function and remove the nuisance factors in this paper. In the final, a numerical example is provided to demonstrate the proposed model and method.

***Keywords**: Proportional intensity model (PIM), imperfect repair model, intensity function, Markov Chain Monte Carlo (MCMC) method, model selection.*

1. Introduction

A repairable system can be defined as a system which is in continuous operation, and which is repaired, but not replaced, after each failure [1]. Intensive research has been performed to address the problem of determining the reliability characteristics of repairable systems, using models based on the Homogeneous Poisson Process (HPP), the Non-Homogeneous Poisson Process (NHPP), and the imperfect repair model *etc.* Such models have been proven successful in engineering. However, the weakness of such models is also criticized by some researchers. As Ascher and Feingold have mentioned, most probabilistic models in the field of reliability have been simplified highly and are based on unrealistic assumptions. Most models consider only one variable, namely the operating time. However, in some situations, the operating time is not the only effective factor influencing reliability, some factors such as the environment, repair history, load *etc*., will also affect reliability greatly [2].

The Proportional Hazard Model (PHM) is one important model which can take the above-mentioned factors into account. The PHM was initially developed for the medical industry [3]. This model comprises two parts. The first part is called the parametric model or baseline function, where the exponential distribution or the Weibull distribution can be used. The second part is called the covariant part. This part accommodates all the covariates, such as the environmental factors, the repair history factors, and so on. Kumar has carried out a thorough survey on the PHM [4, 5]. However, the PHM is suitable for non-repairable systems. When one replace the baseline function with an intensity function, the PHM is extended to the Proportional Intensity Model (PIM), by which the problems of repairable systems can be addressed [6].

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For repairable system, Guo and Love have developed a series of PIMs [7, 8]. These models assume that the system undergoes Preventive Maintenance (PM) and Corrective Operation (CO). After each PM action, the system is assumed to be restored to an as-good-as-new state. Hence the system in each PM periods can be considered as a new identical system. This assumption facilitates parameter estimation for covariates. Based on this assumption, one can use the partial likelihood method to estimate the covariant parameters, regardless of the baseline function [9]. This is one of the significant characteristics of the PHM. However, when no identical systems are observed, this parameter estimate method is not feasible.

In state-of-the-art applications, some NHPP models are usually used as the intensity function in PIM [7, 8]. The NHPP assumes that, after repair, the system is restored to a same-as-old state. Guo *et al.* argue that this assumption is rarely satisfied in practice [10]. In the present paper, we employ the imperfect repair model instead of the NHPP model as the baseline function.

In our paper, we assume that the operating conditions are invariant between each repair and that no PM is carried out on the system. As no identical systems are observed, the partial likelihood estimate method cannot be applied to this case. We propose a Bayesian method to perform parameter estimation. In the Bayesian method, an MCMC method based on slice sampling is used to approximate the posterior distribution.

In the remaining of this paper, Section 2 discusses the model development incorporating imperfect model into PIM. Section 3 discusses the parameter estimate for the developed model. Section 4 presents a numerical example. Section 5 presents the conclusion and discusses the limitation of this proposed model.

Notation

V_n	Virtual age at failure *n*
q	Repair factor
x_n	Inter-arrival time between nth and n-1th failure
λ_0	Intensity function without covariate
λ	Intensity function of PIM model
z	Covariant such as repair history, repair history
γ	Coefficient of covariant
F	Failure probability
R	Reliability
f	Probability density function (PDF)
t_n	The nth Failure time
α	Scale parameter in power law model
β	Shape parameter in power law model
L	Likelihood function
D	Observed failure data
$G(k_1, k_2)$	Gamma distribution with parameter k_1 and k_2
$U(...)$	Uniform distribution

$N(...)$	Normal distribution
$\pi(...)$	Distribution in Bayesian inference
k	Number of parameters in BIC
PIM	Proportional Intensity Model
PHM	Proportional Hazard Model
PM	Preventive Maintenance
CO	Corrective Operation
HPP	Homogeneous Poisson Process
NHPP	Non-Homogeneous Poisson Process
MCMC	Markov Chain Monte Carlo
BIC	Bayesian Information Criterion
OPSK	operator skill
HOILQ	hydraulic oil quality
SCSK	maintenance crew skill
STEMP	hydraulic system temperature
ENDUS	environmental conditions

2. Imperfect Repair Model with Covariates

2.1 Virtual Age Model

It is assumed that the system is rejuvenated after each repair. The effectiveness of each repair is represented by a reduction of the experienced age. Such imperfect repair models are called virtual age models in state-of-the-art research. The Kijima I and Kijima II models are two important representatives of such models [11].

The Kijima I model is described as:

$$V_n = V_{n-1} + qx_n \tag{1}$$

where x_n denotes the inter-arrival time between nth and n-1th failure, V_n and V_{n-1} denotes virtual age at the respective nth and n-1th repair. q is the important imperfect repair factor, which accommodates the degree of repair effectiveness. Usually, q is bounded within a range [0,1].

The corresponding Kijima II model is

$$V_n = q(V_{n-1} + x_n) \tag{2}$$

In both Kijima models, $q = 0$ implies the system has been restored completely. The virtual age model is hence degenerated into a renewal process model. When $q = 1$, the virtual age model is degenerated into a NHPP model.

For the sake of simplicity, we describe the Kijima models as a function:

$$V_n = f(V_{n-1}, x_n) \tag{3}$$

2.2 Proportional Intensity Model with Covariates

Assume the system is experiencing imperfect repair. Let the intensity function of this system under imperfect repair represented by $\lambda_0(t;q)$, the corresponding PIM model represented by $\lambda(t,z;q)$. Similar to the classical Proportional Hazard Model[3], we propose our PIM as follows:

$$\lambda(t,z;q)=\lambda_0(t;q)\exp(\gamma_1 z_1+\gamma_2 z_2+\gamma_3 z_3+...b) \tag{4}$$

For simplicity, we rewrite Formula (4) as:

$$\lambda(t,z;q)=\lambda_0(t;q)\exp\{f(z)\} \tag{5}$$

where $f(z)=\gamma_1 z_1+\gamma_2 z_2+\gamma_3 z_3+...b$.

In this paper, we assume the covariant between each failure are constant and the covariant vary from each failure inter-arrival time. Based on this assumption, the original intensity function, without imperfect repair and with constant operation condition, is shifted horizontally due to imperfect repair. Furthermore, due to the variability of operating conditions, the original intensity function would be shifted vertically. This scenario is illustrated in Figure 1, where the $\lambda(t,z;q)$ is shown not continuous.

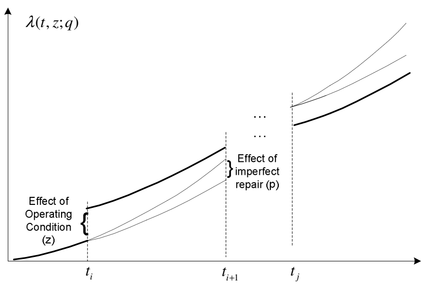

Figure 1: Effect of Imperfect Repair and Operating Conditions

The covariates can be some environmental factors such as temperature, humidity, dust *etc*. Moreover, as Ascher has claimed [2], the repair history can also influence the repair rate. Thus the covariates can be some factors regarding repair history. Percy *et al.* has developed a PIM that considers the repair history factor [12]. In their model, they consider repair history data such as: the time since the last PM, CO, the total number of PM actions and Cos, as covariates. In their paper, the factors regarding repair history are significant. Therefore, considering the influence of the repair history is necessary in some situation. Moreover, the covariates can incorporate some condition monitoring data.

2.3 Cumulative Distribution Function of PIM

Based on the previous failure occurrence, the conditional probability of the next failure can be obtained from [13]:

$$F(x_n/V_{n-1})=\frac{F(x_n+V_{n-1})-F(V_{n-1})}{R(V_{n-1})} \tag{6}$$

It is equivalent to

$$F(x_n/V_{n-1})=\frac{R(V_{n-1})-R(x_n+V_{n-1})}{R(V_{n-1})}=1-\frac{R(x_n+V_{n-1})}{R(V_{n-1})} \tag{7}$$

Substituting Formula (5) into (7), then

$$F(x_n/V_{n-1})=1-e^{-\int_{t_{n-1}}^{t_n}\lambda_0(t;q)\exp\{f(z\}dt} \tag{8}$$

The corresponding Probability Density Function (PDF) is:

$$f(t_n / t_{n-1}) = \lambda_0(t_n)\exp(f(z_n))\exp(-\int_{t_{n-1}}^{t_n} \lambda_0(t)\exp(f(z_n))dt) \quad (9)$$

2.4 PIM under Power Law Process

In this paper, we assume the original intensity function as NHPP. As mentioned before, the NHPP is widely used in modelling repairable systems. NHPP assumes that the successive failures of a system are independent of each other. In other words, the accumulative number of failures has independent increments.

The most popular of NHPP models are Power Law Process Models and Cox-Lewis model [14]. The Power Law Process Model defines the intensity function as a power function and the Cox-Lewis model defines it as a log-linear function. This paper uses the Power Law Process model as original intensity function.

The definition of the Power Law Process model is as follows [15, 16]:

$$\lambda(t) = \frac{\beta}{\alpha}(\frac{t}{\alpha})^{\beta-1} \quad (10)$$

where α is scale parameter, β is shape parameter.

Based on Power Law Process, when the Kijima I model is used to accommodate the effectiveness of repair, the intensity function $\lambda(t, z; q)$ is described as:

$$\lambda(x_i, z; q) = \frac{\beta}{\alpha}(\frac{q\sum_{l=0}^{i-1} x_l + x_i}{\alpha})^{\beta-1}\exp(\gamma_1 z_{1i} + \gamma_2 z_{2i} + \gamma_3 z_{3i} + ...b) \quad (11)$$

where we assume $x_0 = 0$.

When the Kijima II model is used to accommodate the effectiveness of repair, the intensity function $\lambda(t, z; q)$ is:

$$\lambda(x_i, z; q) = \frac{\beta}{\alpha}(\frac{V_{i-1} + x_i}{\alpha})^{\beta-1}\exp(\gamma_1 z_{1i} + \gamma_2 z_{2i} + \gamma_3 z_{3i} + ...b) \quad (12)$$

where V_{i-1} is defined in Equation (2) and we assume $V_0 = 0$.

3. Parameter Estimation and Inference

When the number of covariant considered is large, estimating their corresponding parameter γ_i is also difficult. As there are no identical systems observed, the popular partial likelihood method cannot be applied [17]. In this section, we propose a Bayesian method to estimate parameter.

3.1 Likelihood Function

In this paper, the parameters of interest are $\alpha, \beta, q, \gamma_i$ (i=1,2,…). Assume the failure time is $t_1, t_2, ..., t_n$, the corresponding covariates are $z_1, z_2, ..., z_n$. The PDF based on the intensity function in Formula (11) is then:

$$f(t_n / t_{n-1}) = \frac{\beta}{\alpha}(\frac{x_n + qt_{n-1}}{\alpha})^{\beta-1} \exp(f(z_n)).\exp\left\{(\frac{(qt_{n-1})^{\beta}}{\alpha^{\beta}} - \frac{(x_n + qt_{n-1})^{\beta}}{\alpha^{\beta}})\exp(f(z_n))\right\} \quad (13)$$

The corresponding likelihood function is then rewritten into:

$$L(\alpha, \beta, q, \gamma_i, ...) = \prod_{i=1}^{n} f(t_n / t_{n-1}) \quad (14)$$

When the Kijima I model is used, the corresponding $\ln L$ is:

$$\ln L = n\ln(\frac{\beta}{\alpha}) + \sum_{i=1}^{n} f(z_i) + (\beta-1)\sum_{i=1}^{n}\ln\frac{x_i + qt_{i-1}}{\alpha} + \sum_{i=1}^{n}(-\int_{qt_{n-1}}^{x_n+qt_{n-1}} \beta e^{f(z_i)}(\frac{u}{\alpha})^{\beta-1}du)$$

$$= n\ln\beta - n\beta\ln\alpha + \sum_{i=1}^{n} f(z_i) + (\beta-1)\sum_{i=1}^{n}\ln(x_i + qt_{i-1}) + (\alpha)^{-\beta}\sum_{i=1}^{n} e^{f(z_i)}\{(qt_{i-1})^{\beta} - (qt_{i-1} + x_i)^{\beta}\}$$

$$f(z_i) = \gamma_1 z_{i1} + \gamma_2 z_{i2} + ... \quad (15)$$

The corresponding Kijima II model is similar to above Formula (15), we omit the expression here.

3.2 Bayesian Estimator

When the number of parameters is larger, one can employ Bayesian inference to estimate parameters [19]. The Bayesian estimator considers the parameter as a variable instead of a constant. The Bayesian likelihood function is:

$$L(t) \propto \prod_{i=1}^{n} f(t_n / t_{n-1}) \quad (16)$$

where $f(t_n / t_{n-1})$ is defined in the Formula (13) .

Using the observed “time to failure” data D: $t_1, t_2, ..., t_n$, the joint prior likelihood function is

$$L(D \mid \alpha, \beta, q, \gamma) = \prod_{i=1}^{n} f(t_n / t_{n-1}) \quad (17)$$

One import step to perform Bayesian inference is the selection of the prior distribution for each parameter. We use non-informative priors to estimate parameters in this paper. Similarly to the prior used by Hamada *et al*. [20], we employ a Gamma distribution as the prior distribution for the scale and shape parameters. For the imperfect repair factor q, we use the standard uniform distribution. The prior for the coefficient of the covariate is selected as normal distribution. In summary, the prior distributions are assumed to be: $\alpha \sim G(k_{a1}, k_{\alpha 2})$, $\beta \sim G(k_{\beta 1}, k_{\beta 2})$, $q \sim U(0,1)$, $\gamma \sim N_{\gamma}(\mu_{\gamma}, \sum_{\gamma})$. $G(k_{a1}, k_{\alpha 2})$ is the Gamma distribution with parameters $k_{a1}, k_{\alpha 2}$ for scale parameter α . $G(k_{\beta 1}, k_{\beta 2})$ is that for shape parameter β with parameters $k_{\beta 1}, k_{\beta 2}$. $N_{\gamma}(\mu_{\gamma}, \sum_{\gamma})$ denotes multinomial Normal distribution of covariate coefficient with mean vector μ_{γ} and covariate matrix $\sum_{\gamma}$.

The corresponding joint posterior distributions are

$$\begin{aligned} &\pi(\alpha,\beta,q,\gamma \mid D) \\ &\propto L(D \mid \alpha,\beta,q,\gamma)\times\pi(\alpha \mid k_{a1},k_{\alpha 2}) \\ &\times\pi(\beta \mid k_{\beta 1},k_{\beta 2})\times\pi(q \mid 0,1)\times\pi(\gamma \mid \mu_{\gamma},\textstyle\sum_{\gamma}) \end{aligned} \tag{18}$$

The posterior distribution is a complex compound. We employ an MCMC method with a slice sampler to compute the posterior distribution. One can refer [19] for the theoretical discussion of the MCMC method.

The result of the MCMC estimation will be series of traces. The estimated parameter values of interest are derived from these series of traces. One can refer [20] for the procedure of obtaining the mean, variance and confidence interval from the series of traces.

3.3 Model selection

In Bayesian inference, model selection is a broad term. In this paper, the model selection covers the selection of a suitable baseline function and the removal of nuisance factors. Before we discuss the detailed procedure of model selection, we introduce a method for assessing the performance of the model. The Bayesian Factor is usually used to measure the goodness of models [19]. Yet the Bayesian Factor (BF) is difficult to be obtained practically. When the number of parameters is known, usually an approximate of the BF is used, namely the Bayesian Information Criterion (BIC). The definition of the BIC is as follows:

$$BIC = -2\ln L(D|\alpha,\beta,..) + k\ln n \tag{19}$$

where k is the number of parameters, n is the size of data sets. The models with a lower BIC are more preferable.

In this paper, we firstly employ the Bayesian MCMC method to estimate the parameters considering all covariates and then compute the confidence interval of the estimated parameters. All the covariates whose confidence interval of its γ_i cover zero will be removed. After that, we re-estimate the parameters for the reduced covariates again. This procedure is performed iteratively until no covariate cover zero.

4. A Numerical Example

In order to demonstrate the methodology proposed above, we introduce an example which has been discussed by Ghodrati and Kumar [21]. The hydraulic brake pump is a critical part of the hydraulic loader. It is known that the following factors can influence its reliability: the operator skill (OPSK), maintenance crew skill (SCSK), hydraulic oil quality (HOILQ), hydraulic system temperature (STEMP), and environmental conditions (ENDUS). Our paper uses the data in Table AI from their paper [21].

Several PIM models were applied to model the intensity function considering covariates. They differ at their baseline function: models based on the NHPP and the Weibull distribution, and the Kijima I and Kijima II models, which are imperfect repair models. The Weibull distribution-based model essentially assumed that $q = 0$ and hence assumed that the repair effectiveness restored the system to an as-good-as-new state. The NHPP-based model assumed that the repair effectiveness restored the system to a same-as-old state.

The repair history was also taken into account in the model. The number of repairs experienced was used as a covariate for the PIM, which is abbreviated as CO. In summary, 5 covariates were considered initially. It was assumed that these covariates were independent of each other. We developed our model based on Equation (4). Thereafter we employ Bayesian method to estimate parameter $\alpha, \beta, q, \gamma_i$. The prior distribution was assumed to be: $\alpha \sim G(3,2)$, $\beta \sim G(2,2)$, $q \sim U(0,1)$, $\gamma_i \sim N(0,100)$.

Using MCMC method, after 2000 "burn-in" iterations, 3000 iterations were in a stationary state. The mean, standard deviation, and confidence interval for the parameters of interest were derived from the traces which are in stationary state. The parameters with a confidence interval covering zero were considered insignificant. After removing all the insignificant factors, we re-evaluated the parameter. All the confidence intervals of the remaining covariates excluded zero. The results using different models are tabulated in Table 1.

Table 1: Results using Several Models

Baseline Function	Model	Covariates	BIC
NHPP	No covariate	/	104
	Full covariates	$\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5\ \gamma_6$	111
	Reduced covariates	$\gamma_1\ \gamma_2\ \gamma_3\ \gamma_6$	106.7
Kijima I	No covariate	/	120
	Full covariates	$\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5\ \gamma_6$	62
	Reduced covariates	$\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_6$	59.66
Kijima II	No covariate	/	95.6
	Full covariates	$\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5\ \gamma_6$	57
	Reduced covariates	$\gamma_1\ \gamma_2\ \gamma_4$	54
Weibull Distribution	No covariate	/	292
	Full covariates	$\gamma_1\ \gamma_2\ \gamma_3\ \gamma_4\ \gamma_5\ \gamma_6$	63.36
	Reduced covariates	$\gamma_1\ \gamma_2\ \gamma_4\ \gamma_6$	58.92

From Table 1, it shows that the NHPP-based series of models and models without covariates are not suitable for the data, as the BICs are all high. Therefore, the assumption of restoration to a same-as-old state is not reasonable and some covariates should be incorporated into the model. The Kijima I and the Weibull distribution-based models exhibit a similar performance in this application. The reason behind is that, when the Kijima I model is used, q is near zero. Therefore, the effectiveness of q can be considered as restoring the system to an as-good–as-new state.

The best model is the Kijima II based PIM model with the covariates $\gamma_1\ \gamma_2\ \gamma_4$, corresponding to BIC=54. The detailed results for this model are tabulated in Table 2 where SD is standard deviation.

Table 2: Evaluated parameter values

Parameter	Mean	SD	Lower bound (0.025)	Upper bound (0.975)
Scale Parameter α	3.7878	0.2833	3.2936	4.41
Shape Parameter β	7.83	1.29	5.5	10.6
Imperfect Repair Factor q	0.099	0.0394	0.0260	0.1838
OPSK β_1	-2.584	0.6212	-3.81	-1.55
SCSK β_2	-0.978	0.3722	-1.71	-0.27
STEMP β_4	-1.51	0.4155	-2.3538	-0.6831

In the optimal PIM model, the mean for q is 0.099. In order to demonstrate the effectiveness of repair, we plot its intensity function against time with $q = 0.099$, as shown in Figure 2, where the intensity function is for the 14th to the 17th failure. We can see from this figure that repair is significantly effective as the system's intensity function has almost been restored to zero after each repair.

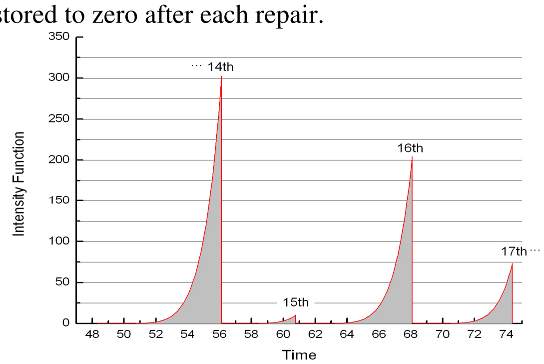

Figure 2: Effectiveness of Repair

Moreover, in the optimal PIM model, the remaining significant factors are: OPSK, SCSK and STEMP, which are all significant at level 0.05. To illustrate the effectiveness of these factors, we take the factor STEMP as an example. Figure 3 plots the 95% confidence interval of the intensity function, when STEMP is 1 and the other covariates are zero. Both the upper bound and the lower bound are under the "No Covariate" curve. It implies that under the higher STEMP, the system has higher reliability than the normal STEMP.

Additionally, in order to compare the effectiveness of all 3 factors on the system reliability, Figure 4 is plotted. In Figure 4, intensity function "OPSK" supposes the OPSK to be 1 and the other factors are 0. The other curves for factor STEMP, SCSK follow this. This figure shows the effectiveness of the 3 factors on reliability follows the order: OPSK> STEMP > SCSK. The factor OPSK influences the system reliability most significantly.

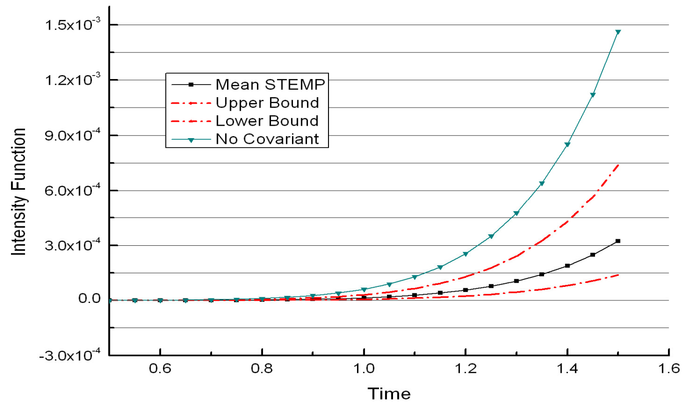

Figure 3: 95% Confidence Interval for Intensity Function

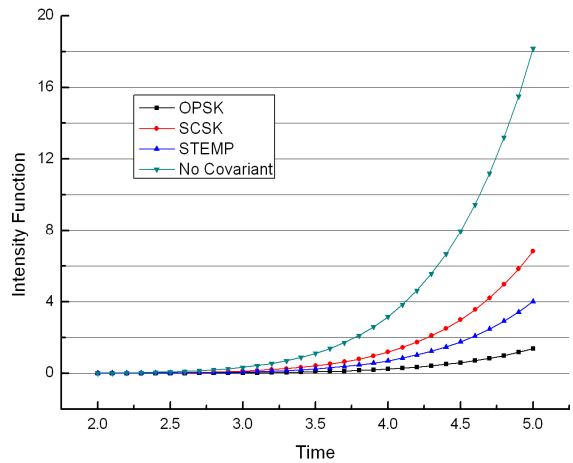

Figure 4: Sensitivity of Significant Factors

CO, which represents the repair history in this example, is insignificant in this optimal PIM model. This does not mean that the repair history can be neglected. In some other situations, considering the repair history is still necessary.

Finally, it is necessary to mention that Liao has developed an imperfect repair model that considers the cumulative number of failures as a covariate [22]. He argues that his imperfect repair model can outperform any other imperfect repair model. Essentially, the imperfect repair PIM model proposed in our paper has generalized that presented in Liao's paper. In our model, not only the cumulative number of failures, but also any factors relevant to failure can be accommodated as covariates.

5. Conclusions

This paper has proposed a model that combines the imperfect repair model and the proportional intensity model. Using this proposed model, the effectiveness of repair and covariates are incorporated into the model. The paper essentially provides a framework for accommodating all possible factors into a model to analyze their effectiveness. These factors could be the operating conditions, the environmental fluctuation, and the repair or maintenance history *etc*. The introduction of a large number of factors into the model complicates the estimation of parameters. In contrast to some PIMs considering several identical systems existing, where the partial likelihood estimator can be used, the paper employs a Bayesian inference method based on MCMC estimation. This parameter estimator does not require identical systems.

One limitation of the present research is the fact that the factors are assumed to be mutually independent. In practice, especially when the repair history factor is considered, some factors can be highly correlated. The interaction between factors should therefore be considered. Another limitation of this paper is the fact that the covariates are assumed to be time-independent. Further research will extend this proposed model to consider time-dependent factors.

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