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Bayesian semi-parametric analysis for locomotive wheel degradation using gamma frailties

Jing Lin[1,2] and Matthias Asplund[1,2,3]

Abstract

A reliability study based on a Bayesian semi-parametric framework is performed in order to explore the impact of the position of a locomotive wheel on its service lifetime and to predict its other reliability characteristics. A piecewise constant hazard regression model is used to analyse the lifetime of locomotive wheels using degradation data and taking into account the bogie on which the wheel is located. Gamma frailties are included in this study to explore unobserved covariates within the same group. The goal is to flexibly determine reliability for the wheel. A case study is performed using Markov chain Monte Carlo methods and the following conclusions are drawn. First, a polynomial degradation path is a better choice for the studied locomotive wheels; second, under given operational conditions, the position of the locomotive wheel, i.e. on which bogie it is mounted, can influence its reliability; third, a piecewise constant hazard regression model can be used to undertake reliability studies; fourth, considering gamma frailties is useful for exploring the influence of unobserved covariates; and fifth, the wheels have a higher failure risk after running a threshold distance, a finding which could be applied in optimisation of maintenance activities.

Keywords

Reliability, Bayesian survival analysis, locomotive wheels, frailty, piecewise constant hazard rate, Markov chain Monte Carlo

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Introduction

The service life of a train wheel can be significantly reduced due to failure or damage, leading to excessive cost and accelerated deterioration, a point which has received considerable attention in recent literature. In order to monitor the performance of wheel-sets and make replacements in a timely fashion, the railway industry uses both preventive and predictive maintenance. By predicting the wear of train wheels[1–3], fatigue[4,5], tribology aspects[6] and failures[7], the industry can design strategies for different types of preventive maintenance (reprofiling, lubrication, etc.) for various periods (days, months, seasons, running distance, etc.). Software dedicated to predicting wear rate has also been proposed.[8] Finally, condition monitoring data have been studied with a view to increasing a wheel's lifetime.[9–12]

One common preventive maintenance strategy (used in the case study) is to reprofile wheels after they run a certain distance. Reprofiling affects the wheel's diameter; once the diameter is reduced to a pre-specified length, the wheel is replaced by a new one. Seeking to optimise this maintenance strategy, researchers have examined wheel degradation data to determine wheel reliability and failure distribution (see Freitas et al.[13] and Freitas et al.[14] and the references therein). However, these studies only selected the data that was most of interest, without simultaneously considering the influence of several covariates (e.g. the wheel's installed position). For example, Freitas et al.[13] and Freitas et al. [14] studied the diameter measurements performed for the wheels of 14 trains (each one composed of four cars: CA1, CA2, CA3, CA4). To avoid the potential influence of wheel location, they only considered those on the left side of axle number 1 of each one of the CA1 cars, but pointed out that 'the degradation of a given wheel might be associated with its position on a given car'. Yang and Letourneau[7] suggested that certain attributes, including a wheel's installed position (right or left), might influence its

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wear rate, but they did not provide case studies. Palo et al.[15] concluded that 'different wheel positions in a bogie show significantly different force signatures'. To address the noted issues, Lin et al.[16] have explored the influence of the positioning of locomotive wheels on their reliability. Their results indicate that the particular bogie on which the wheel is mounted has a greater influence on its lifetime than does the axle or which side it is on. Therefore, in this paper, we only use the bogie as a covariate.

Most reliability studies are implemented under the assumption that individual lifetimes are independent identically distributed (IID). However, sometimes Cox proportional hazard (CPH) models cannot be used because of the dependence of data within a group. For instance, because they have the same operating conditions, the wheels mounted on a particular locomotive may be dependent. In a different context, some data may come from multiple records which actually belong to the wheels installed in the same position but on a different locomotive. Modelling dependence in multivariate survival data has received considerable attention, particularly for cases where the datasets may come from subjects of the same group which are related to each other.[17,18] A key development in modelling such data is to consider frailty models, in which the data are conditionally independent. When frailties are considered, the dependence within subgroups can be considered an unknown and unobservable risk factor (or explanatory variable) of the hazard function. In this paper, wheels installed on the same locomotive are considered as coming from one group; among them, wheels installed on the same bogie of one locomotive, are considered as one subgroup. In addition, we consider a gamma shared frailty, first discussed by Clayton[19] and Oakes[20] and later developed by Sahu et al.[17], to explore the unobserved covariates' influence on the wheels on the same locomotive.

In addition, since semi-parametric Bayesian methods offer a more general modelling strategy that contains fewer assumptions than directly adopting parametric distributions[21], we adopt the piecewise constant hazard model to analyse the distribution of the locomotive wheels' lifetime. The applied hazard function is sometimes referred to as a piecewise exponential model; it is convenient because it can accommodate various shapes of the baseline hazard over the intervals.

This paper explores the impact of a locomotive wheel's installed position on its service lifetime and predicts its other reliability characteristics by using a Bayesian semi-parametric framework. The remainder of this paper is organised as follows. The piecewise constant hazard regression model with gamma frailties is discussed in the next section. In the proposed model, a discrete-time Martingale process is considered as a prior process for the baseline hazard rate. The section 'Case study' describes a real case study using a dataset for the wheels of two locomotives powering a heavy haul cargo train. Using polynomial degradation, it considers the bogies as covariates and uses a gamma frailty for each locomotive. It adopts a Markov chain Monte Carlo (MCMC) computational scheme[22,23] and discusses strategies for optimisation of maintenance activities. Finally, conclusions are drawn and directions for future study are proposed.

Bayesian semi-parametric models using the gamma frailty

In this section, we propose a Bayesian semi-parametric framework, incorporating the piecewise constant hazard regression model, a gamma shared frailty model, the discrete-dime Martingale process for the baseline hazard rate, and a MCMC computation scheme.

Piecewise constant hazard regression model

The piecewise constant hazard model is one of the most convenient and popular semi-parametric models in survival analysis. Begin by denoting the jth individual in the ith group as having lifetime t_{ij}, where $i = 1, \ldots, n$ and $j = 1, \ldots, m_i$. Divide the time axis into intervals $0 < s_1 < s_2 < \cdots < s_k < \infty$, where $s_k > t_{ij}$, thereby obtaining k intervals $(0, s_1]$, $(s_1, s_2], \ldots, (s_{k-1}, s_k]$. Suppose that the jth individual in the ith group has a constant baseline hazard $h_0(t_{ij}) = \lambda_k$ as in the kth interval, where $t_{ij} \in I_k = (s_{k-1}, s_k]$. Then, the hazard rate function for the piecewise constant hazard model can be written as

$$h_0(t_{ij}) = \lambda_k, \quad t_{ij} \in I_k \tag{1}$$

Equation (1) is sometimes referred to as a piecewise exponential model; it can accommodate various shapes of the baseline hazard over the intervals.

Studies of how to divide the time axis into k intervals include the following. Kalbfleisch and Prentice[24] suggested that the selection of intervals should be made independent of the data; this has been adopted in the construction of the traditional lifetime table. Breslow[25] suggested using distinct failure times as end points of each interval. Both Sahu et al.[17] and Ibrahim et al.[21] pointed out that the choice of a large value for k will make the model non-parametric, however, too large a k will produce unstable estimators of the λ values and too small a choice will lead to poor model fitting. Therefore, they discussed the robust choice of k by considering a correlated process prior. Aslanidou et al.[18] suggested that 'for many practical situations, the grid will be created using intervals of equal length', but they also indicated that 'one may also assume a grid of irregular intervals' since 'the choice of k is up to the statistician and this model may be sufficiently flexible to offer a satisfactory practical approximation'. Suppose $\mathbf{x}_i = (x_{1i}, \ldots, x_{pi})'$ denotes the covariate vector for the individuals in the ith group, and $\boldsymbol{\beta}$ is the regression parameter. Therefore, the regression model with the piecewise constant hazard rate can be written as

$$h(t_{ij}) = \begin{cases} \lambda_1 \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}), & 0 < t_{ij} \leqslant s_1 \\ \lambda_2 \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}), & s_1 < t_{ij} \leqslant s_2 \\ \vdots & \vdots \\ \lambda_k \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}), & s_{k-1} < t_{ij} \leqslant s_k \end{cases} \tag{2}$$

Correspondingly, its probability density function $f(t_{ij})$, cumulative distribution function $F(t_{ij})$, reliability function $R(t_{ij})$, together with the cumulative hazard rate $\Lambda(t_{ij})$ can be achieved.[21]

Gamma shared frailty model. Frailty models were first considered by Clayton[19] and Oakes[20] to handle multivariate survival data. In their models, the event times are conditionally independent according to a given frailty factor, which is an individual random effect. As discussed by Sahu et al.[17], the models formulate different variability factors that come from two distinct sources. The first source is natural variability, which is explained by the hazard function; the second is variability common to individuals of the same group or variability common to several events of an individual, which is explained by the frailty.

Assume the hazard function for the jth individual in the ith group is

$$h_{ij}(t) = h_0(t)\omega_i \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}) \tag{3}$$

In equation (3), which is a multiplicative frailty model, ω_i represents the frailty parameter for the ith group. In this equation ω_i is shared by individuals in the same group, and it is thus referred to as the shared frailty model and actually is an extension of the CPH model (as mentioned in Aslanidou et al.[18], frailty models first arose because some researchers extended the CPH model by including a random effect or frailty for the group effect).

To date, discussions on frailty models have focused on the choices of either the form of the baseline hazard function or the form of the frailty's distribution. Representative studies related to the former include the gamma process for the accumulated hazard function[26,27], Weibull baseline hazard rate[17] and the piecewise constant hazard rate[18] which is adopted in this paper due to its flexibility.[21] Some researchers have examined finite mean frailty distributions, including gamma distribution[19,28], lognormal distribution[29], etc.; others have studied non-parametric methods, including the inverse Gaussian frailty distribution[30], the power variance function for frailty[31], the positive stable frailty distribution[32,33], the Dirichlet process frailty model[34] and the Levy process frailty model.[35] In this paper, we consider the gamma shared frailty model, the most popular model for frailty in current application studies.[21]

Suppose the frailty parameters ω_i are IID for each group, and follow a gamma distribution, denoted by $Ga(\kappa^{-1}, \kappa^{-1})$. Therefore, the probability density function can be written as

$$f(\omega_i) = \frac{(\kappa^{-1})^{\kappa^{-1}}}{\Gamma(\kappa^{-1})} \times \omega_i^{\kappa^{-1}-1} \exp(-\kappa^{-1}\omega_i) \tag{4}$$

In equation (4), the mean value of ω_i is one, and κ is the unknown variance of the values of ω_i. Greater values of κ signify a closer positive relationship between the subjects of the same group as well as greater heterogeneity among groups. Furthermore, as $\omega_i > 1$, the failures for the individuals in the corresponding group will appear earlier than if $\omega_i = 1$; in other words, as $\omega_i < 1$, their predicted lifetimes will be greater than those found in the independent models.

Suppose $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_n)'$, considering equation (5), then the likelihood function

$$\pi(\boldsymbol{\omega}|\kappa) = \prod_{i=1}^{n} f(\omega_i|\kappa) \propto \prod_{i=1}^{n} \omega_i^{\kappa^{-1}-1} \exp(-\kappa^{-1}\omega_i) \tag{5}$$

Equation (5) will be used to achieve joint posterior density in equations (10) to (14).

Discrete-time Martingale process for baseline hazard rate

Based on the presented discussions (equations (2), (3) and (4)), the piecewise constant hazard model with gamma shared frailties can be written as

$$h(t_{ij}) = \begin{cases} \lambda_1\omega_i \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}), & 0 < t_{ij} \leqslant s_1 \\ \lambda_2\omega_i \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}), & s_1 < t_{ij} \leqslant s_2 \\ \vdots & \vdots \\ \lambda_k\omega_i \exp(\mathbf{x}_{ij}'\boldsymbol{\beta}), & s_{k-1} < t_{ij} \leqslant s_k \end{cases} \tag{6}$$

To analyse the baseline hazard rate λ_k, a common choice is to construct an independent incremental process, e.g. the gamma process, the beta process or the Dirichlet process. In this paper, the discrete-time Martingale process for the baseline hazard rate λ_k, which is discussed by Sahu et al.[17] and Aslanidou et al.[18], is adopted.

Given $(\lambda_1, \lambda_2, \ldots, \lambda_{k-1})$, we specify that

$$\lambda_k|\lambda_1, \lambda_2, \ldots, \lambda_{k-1} \sim Ga\left(\alpha_k, \frac{\alpha_k}{\lambda_{k-1}}\right) \tag{7}$$

Let $\lambda_0 = 1$. In equation (7), the parameter α_k represents the smoothness for the prior information. If $\alpha_k = 0$, then λ_k and λ_{k-1} are independent. As $\alpha_k \to \infty$, the baseline hazard is the same in nearby intervals. In addition, the Martingale λ_k's expected value at any time point is the same, and

$$E(\lambda_k|\lambda_1, \lambda_2, \ldots, \lambda_{k-1}) = \lambda_{k-1} \tag{8}$$

Equation (8) shows that given specified historical information $(\lambda_1, \lambda_2, \ldots, \lambda_{k-1})$, the expected value of λ_k is fixed.

Bayesian semi-parametric model using MCMC

In reliability analysis, the lifetime data are usually incomplete, and only a portion of the individual lifetimes are known. Right-censored data are often called Type I censoring, and the corresponding likelihood construction problem has been extensively studied in the literature.[36,37] Suppose that the jth individual in the ith group has lifetime T_{ij} and censoring time L_{ij}. The observed lifetime $t_{ij} = \min(T_{ij}, L_{ij})$; therefore, the exact lifetime T_{ij} will be observed only if $T_{ij} \leqslant L_{ij}$. In addition, the lifetime data involving right-censoring can be represented by n pairs of random variables (t_{ij}, υ_{ij}), where $\upsilon_{ij} = 1$ if $T_{ij} \leqslant L_{ij}$ and $\upsilon_{ij} = 0$ if $T_{ij} > L_{ij}$. This means that υ_{ij} indicates whether lifetime T_{ij} is censored or not. The likelihood function is deduced as

$$L(t) = \prod_{i=1}^{n} \prod_{j=1}^{m_i} [f(t_{ij})]^{\upsilon_{ij}} R(t_{ij})^{1-\upsilon_{ij}} \tag{9}$$

In the previously presented piecewise constant hazard model, we denote g_{ij} as $t_{ij} \in (s_{g_{ij}}, s_{g_{ij}+1}) = I_{g_{ij}+1}$ and the model's dataset as $D = (\omega, \mathbf{t}, \mathbf{X}, \upsilon)$. Following equations (7) to (10), the complete likelihood function $L(\boldsymbol{\beta}, \lambda|D)$ for the individuals for the ith group in k intervals can be written as

$$\prod_{i=1}^{n} \prod_{j=1}^{m_i} \left\{ \begin{array}{l} [\prod_{k=1}^{g_{ij}} \exp(-\lambda_k \omega_i \exp(\mathbf{x}'_{ij}\boldsymbol{\beta})(s_k - s_{k-1})] \\ \times(\lambda_{g_{ij}+1} \omega_i \exp(\mathbf{x}'_{ij}\boldsymbol{\beta}))^{\upsilon_{ij}} \\ \times \exp[-\lambda_{g_{ij}+1} \omega_i \exp(\mathbf{x}'_{ij}\boldsymbol{\beta})(t_{ij} - s_{g_{ij}})] \end{array} \right\} \tag{10}$$

Let $\pi(\cdot)$ denote the prior or posterior distributions for the parameters. Following equations (5) and (10), the joint posterior distribution $\pi(\omega_i|\boldsymbol{\beta}, \lambda, D)$ for gamma frailties ω_i can be written as

$$\begin{aligned} \pi(\omega_i|\boldsymbol{\beta}, \lambda, D) &\propto L(\boldsymbol{\beta}, \lambda|D) \times \pi(\omega|\kappa) \\ &\propto \omega_i^{\kappa^{-1}+\sum_{j=1}^{m_i} \upsilon_{ij}-1} \exp\left\{-\left(\kappa^{-1} + \left[\sum_{j=1}^{m_i} \exp(\mathbf{x}'_{ij}\boldsymbol{\beta})\right]\right)\right. \\ &\quad \left.\times\left(\sum_{k=1}^{g_{ij}} \lambda_k(s_k - s_{k-1}) + \lambda_{g_{ij}+1}(t_{ij} - s_{g_{ij}})\right)\right\} \\ &\sim \tilde{G}a\left\{\kappa^{-1} + \sum_{j=1}^{m_i} \upsilon_{ij}, \kappa^{-1} + \left[\sum_{j=1}^{m_i} \exp\left(\mathbf{x}'_{ij}\boldsymbol{\beta}\right)\right]\right. \\ &\quad \left.\times\left(\sum_{k=1}^{g_{ij}} \lambda_k(s_k - s_{k-1}) + \lambda_{g_{ij}+1}(t_{ij} - s_{g_{ij}})\right)\right\} \end{aligned} \tag{11}$$

Equation (11) shows that the full conditional density of each ω_i is a gamma distribution. Similarly, the full conditional density of κ^{-1} and $\boldsymbol{\beta}$ can be given by

$$\begin{aligned} \pi(\kappa^{-1}|\boldsymbol{\beta}, \boldsymbol{\omega}, \lambda, D) &\propto \prod_{i=1}^{n} \omega_i^{\kappa^{-1}-1} (\kappa^{-1})^{-n\kappa^{-1}} \\ &\quad \times \frac{\exp(-\kappa^{-1} \sum_{i=1}^{n} \omega_i)}{[\Gamma(\kappa^{-1})]^n} \times \pi(\kappa^{-1}) \end{aligned} \tag{12}$$

$$\begin{aligned} \pi(\boldsymbol{\beta}|\kappa^{-1}, \boldsymbol{\omega}, \lambda, D) &\propto \exp\left\{\sum_{i=1}^{n} \sum_{j=1}^{m_i} \upsilon_{ij} \mathbf{x}'_{ij}\boldsymbol{\beta} - \sum_{i=1}^{n} \sum_{m=1}^{n_i}\right. \\ &\quad \times \exp\left(\mathbf{x}'_{ij}\boldsymbol{\beta}\right)\omega_i \times \left[\sum_{k=1}^{g_{ij}} \lambda_k(s_k - s_{k-1})\right. \\ &\quad \left.\left. + \lambda_{g_{ij}+1}(t_{ij} - s_{g_{ij}})\right]\right\} \times \pi(\beta) \end{aligned} \tag{13}$$

Let $R_k = \{(i,j);\, t_{ij} > s_k\}$ denote the risk set at s_k and $D_k = R_{k-1} - R_k$; let d_k denote the failure individuals in the interval I_k. Let $\pi(\lambda_k|\lambda^{(-k)})$ denote the conditional prior distribution for $(\lambda_1, \lambda_2, \ldots, \lambda_J)$ without λ_k. We therefore derive $\pi(\lambda_k|\boldsymbol{\beta}, \boldsymbol{\omega}, \kappa^{-1}, D)$ as

$$\lambda_k^{d_k} \exp\left\{ \begin{array}{l} -\lambda_k \omega_i \exp(\mathbf{x}'_{ij}\boldsymbol{\beta}) \times \left[\sum_{(i,j)\in R_k} (s_k - s_{k-1}) + \right. \\ \left. + \sum_{(i,j)\in D_k} (t_{ij} - s_{k-1}) \right] \end{array} \right\} \times \pi(\lambda_k|\lambda^{(-k)}) \tag{14}$$

Case study

In this section, we present a case study to illustrate the use of the proposed Bayesian semi-parametric models for the degradation analysis of locomotive wheels.

Degradation data

The data were collected between November 2010 and January 2012 by a Swedish railway company. We use the degradation data from two heavy haul cargo trains' locomotives (denoted as locomotive 1 and

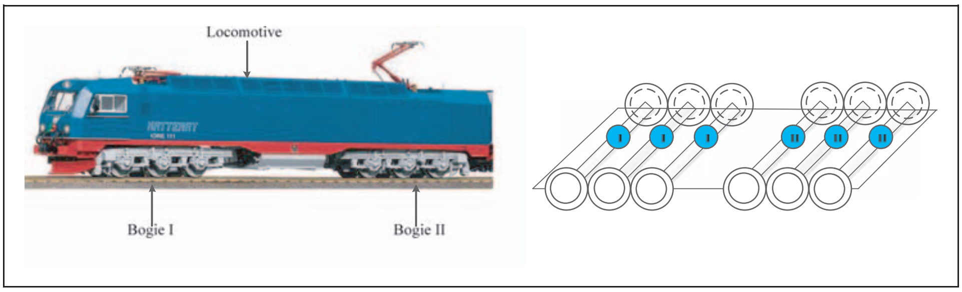

Figure 1. Wheel positions specified in this study.

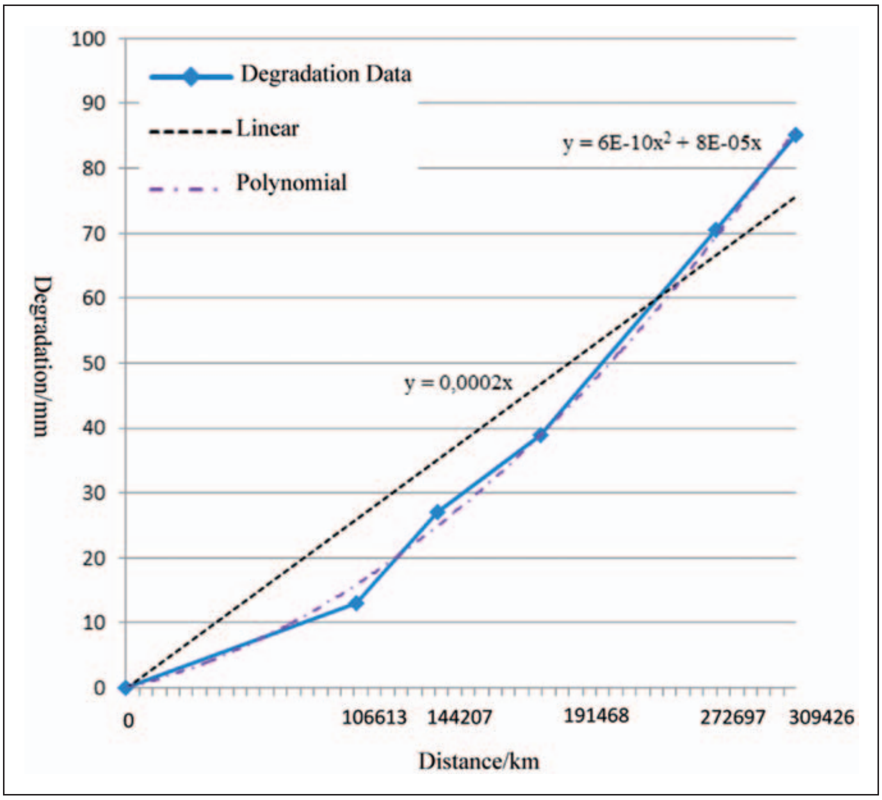

Figure 2. Plot of the wheel degradation data: an example.

locomotive 2). Correspondingly, there are two studied groups, and $n = 2$. For each locomotive, see Figure 1, there are two bogies (bogie I, bogie II), and for each bogie, there are six wheels. The installed positions of the wheels on a particular locomotive are specified by the bogie number and are defined as covariates **x**. The covariates' coefficients are represented by **β**. More specifically, $x = 1$ represents the wheels mounted on bogie I, and $x = 2$ represents the wheels mounted in bogie II. β_1 is the coefficient of the covariates **x**, and β_0 is defined as natural variability.

The diameter of a new locomotive wheel is 1250 mm. In the company's current maintenance strategy, a wheel's diameter is measured after running a certain distance. If it has been reduced to 1150 mm, the wheel is replaced by a new one. Otherwise, it is reprofiled or other maintenance strategies are implemented. Therefore, a threshold level for failure, denoted as y_0, is defined as 100 mm ($y_0 = 1250 - 1150$ mm). The wheel's failure condition is assumed to be reached if the diameter reaches y_0. The company's complete dataset includes the diameters of all locomotive wheels at a given inspection time, the total running distances corresponding to their 'time to be maintained (reprofiled or replaced)', and the wheels' bill of material data, from which we can determine their positions.

Degradation path and lifetime data

From the dataset, we obtain five or six measurements of the diameter of each wheel during its lifetime. By connecting these measurements, we can determine a degradation trend (e.g. in Figure 2, the blue line).

In their analyses of train wheels, most studies assume a linear degradation path (see the black dotted line in Figure 2). However, in our study, the results show that a better choice is a polynomial degradation path (see Figure 2, the purple dotted line). We plot the degradation data for one locomotive wheel in Figure 2. The squares of their correlation coefficients (denoted as R^2) are 0.9973 for a polynomial path and 0.9271 for a linear path, indicating that a polynomial degradation path is better than a linear degradation path for the wheels studied.

Compared with the linear degradation path, the polynomial degradation path in this study suggests the degradation rate of the wheels is increasing. One possible improvement for maintenance is to remove more material at later reprofiling because the depth at an earlier reprofiling was inadequate to remove the full depth of surface damage. However, mismatched diameters (from machining tolerances) because of additional wear could also be a possibility.

Take locomotive 1 for example; for all wheels installed on this locomotive, the assumptions of the polynomial function are supported by the statistics shown in Table 1. Note: some lifetime data are right-censored (denoted by superscript * in Table 1). However, we know the real lifetimes will exceed the predicted lifetimes.[16] Considering the likelihood function, if those data are recognised as completed data, it is clear that some information will be neglected and the results will be underestimated. Table 2 shows the results of the same test for the wheels on locomotive 2. Again, a polynomial degradation path is a better choice.

Following the presented discussion, a wheel's failure condition was assumed to be reached if the

Table 1. Statistics on degradation path and lifetime data: locomotive 1.

Number	Positions	Polynomial path	R^2-polynomial	Linear path	R^2-linear	Lifetime ($\times$1000 km)
1	Bogie I	$y = 6E\text{-}10x^2 + 8E\text{-}05x$	0.9973	$y = 0.0002x$	0.9271	347
2	Bogie I	$y = 6E\text{-}10x^2 + 8E\text{-}05x$	0.9974	$y = 0.0002x$	0.9274	347
3	Bogie I	$y = 7E\text{-}10x^2 + 6E\text{-}05x$	0.9981	$y = 0.0002x$	0.9109	338
4	Bogie I	$y = 7E\text{-}10x^2 + 6E\text{-}05x$	0.9982	$y = 0.0002x$	0.9102	338
5	Bogie I	$y = 6E\text{-}10x^2 + 7E\text{-}05x$	0.9986	$y = 0.0002x$	0.9211	354
6	Bogie I	$y = 6E\text{-}10x^2 + 7E\text{-}05x$	0.9986	$y = 0.0002x$	0.9215	354
7	Bogie II	$y = 1E\text{-}09x^2 + 4E\text{-}06x$	0.9960	$y = 0.0002x$	0.8485	314*
8	Bogie II	$y = 1E\text{-}09x^2 + 4E\text{-}06x$	0.9960	$y = 0.0002x$	0.8485	314*
9	Bogie II	$y = 1E\text{-}09x^2 + 4E\text{-}06x$	0.9964	$y = 0.0002x$	0.8419	314*
10	Bogie II	$y = 1E\text{-}09x^2 + 3E\text{-}06x$	0.9963	$y = 0.0002x$	0.8430	315*
11	Bogie II	$y = 7E\text{-}10x^2 + 7E\text{-}05x$	0.9792	$y = 0.0002x$	0.9039	331
12	Bogie II	$y = 7E\text{-}10x^2 + 7E\text{-}05x$	0.9805	$y = 0.0003x$	0.9027	331

*Right-censored data

Table 2. Statistics on degradation path and lifetime data: locomotive 2.

Number	Positions	Polynomial path	R^2-polynomial	Linear path	R^2-linear	Lifetime ($\times$1000 km)
1	Bogie I	$y = 8E\text{-}10x^2 + 0.0002x$	0.9807	$y = 0.0003x$	0.9579	250
2	Bogie I	$y = 8E\text{-}10x^2 + 0.0002x$	0.9817	$y = 0.0003x$	0.9597	250
3	Bogie I	$y = 8E\text{-}10x^2 + 0.0002x$	0.9805	$y = 0.0003x$	0.9590	250
4	Bogie I	$y = 8E\text{-}10x^2 + 0.0002x$	0.9798	$y = 0.0003x$	0.9589	250
5	Bogie I	$y = 7E\text{-}10x^2 + 0.0002x$	0.9790	$y = 0.0003x$	0.9624	261
6	Bogie I	$y = 7E\text{-}10x^2 + 0.0002x$	0.9792	$y = 0.0003x$	0.9624	261
7	Bogie II	$y = 9E\text{-}10x^2 + 0.0002x$	0.9751	$y = 0.0003x$	0.9491	240
8	Bogie II	$y = 1E\text{-}09x^2 + 0.0002x$	0.9750	$y = 0.0003x$	0.9484	232
9	Bogie II	$y = 1E\text{-}09x^2 + 0.0002x$	0.9709	$y = 0.0003x$	0.9420	232
10	Bogie II	$y = 1E\text{-}09x^2 + 0.0002x$	0.9689	$y = 0.0003x$	0.9415	232
11	Bogie II	$y = 9E\text{-}10x^2 + 0.0002x$	0.9727	$y = 0.0003x$	0.9498	240
12	Bogie II	$y = 9E\text{-}10x^2 + 0.0002x$	0.9726	$y = 0.0003x$	0.9495	240

diameter reaches y_0 ($y_0 = 100\,\text{mm}$). We adopted the polynomial path for all wheels and set $y_0 = y$ (y represent the function of the degradation path $y(x)$, see Table 1). The lifetimes for these wheels were easily determined and are shown in the last columns of Table 1 and Table 2.

Table 3. Posterior distribution summaries.

Parameter	Mean	SD	MC error	95% HPD interval
β_0	−10.39	2.888	0.2622	(−16.61, −4.79)
β_1	0.3293	0.4927	0.02016	(−0.661, 1.271)
κ	0.563	0.269	0.01038	(0.1879, 1.225)
ω_1	0.1441	0.1374	0.004822	(0.01192, 0.5258)
ω_2	1.866	1.016	0.03628	(0.3846, 4.308)
b_1	0.1361	1.595	0.1037	(−3.196, 3.364)
b_2	0.758	2.182	0.1672	(−3.7, 5.248)
b_3	1.94	2.514	0.2105	(−3.126, 7.342)
b_4	4.447	2.668	0.2389	(−0.5652, 10.48)
b_5	6.342	2.684	0.2415	(1.126, 12.29)
b_6	8.159	2.724	0.2417	(2.843, 14.15)

Parameter configuration

Sahu et al.[17] and Ibrahim et al.[21] pointed out that a choice of a very large k will make the model non-parametric, however, too large a k will produce unstable estimators of the λ values and too small a value will lead to poor model fitting. In our study, the degradation path is obtained from analysis of five or six measurements for each locomotive wheel; in other words, the lifetime data are based on the data acquired at five or six different inspections. For ease of calculation, in this paper, we adopted a very common choice for many practical situations[18]: that of creating intervals of equal length and dividing the time axis into six sections piecewise. In our case study, no predicted lifetime exceeds 360,000 km. Therefore, $k = 6$, and each interval is equal to 60,000 km. We obtained six intervals (0, 60,000], (60,000, 120,000], . . . , (300,000, 360,000]. (Note that, since the data in this case shows there are no failures before

Table 4. Baseline hazard rate statistics.

Piecewise intervals (×1000 km)	1	2	3	4	5	6
	(0, 60]	(60, 120]	(120, 180]	(180, 240]	(240, 300]	(300, 360]
λ_k	1.15	2.13	6.96	85.37	567.93	3494.69

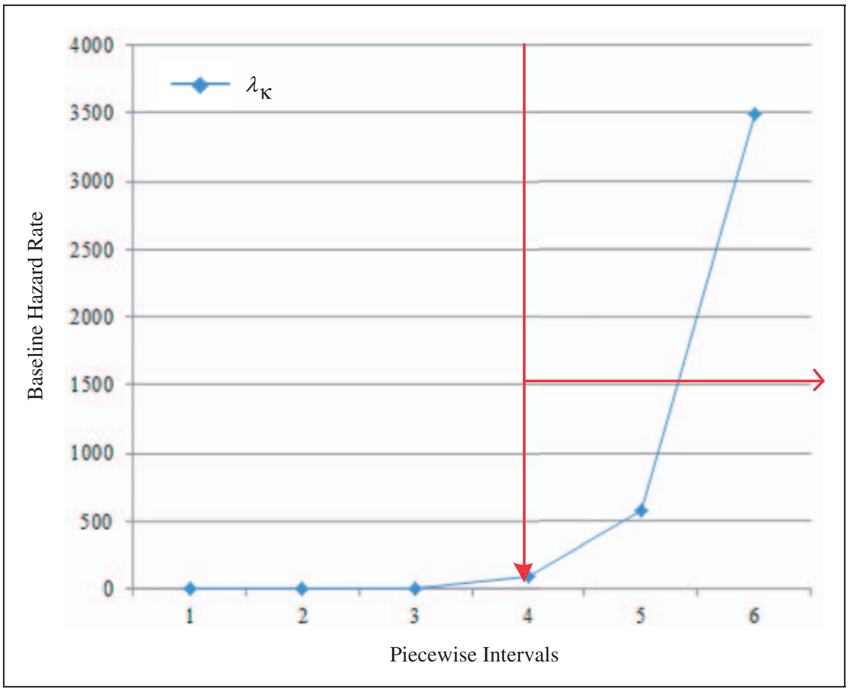

Figure 3. Plot of baseline hazard rate.

Table 5. Reliability and cumulative hazard statistics.

Distance (1000 km)	Reliability $R(t)$				Cumulative hazard $\Lambda(t)$			
	Locomotive 1		Locomotive 2		Locomotive 1		Locomotive 2	
	Bogie I	Bogie II	Bogie I	Bogie II	Bogie I	Bogie II	Bogie I	Bogie II
60	0.999,577	0.999,412	0.994,534	0.992,41	0.000,184	0.000,256	0.002,38	0.003,309
120	0.998,425	0.997,811	0.979,79	0.972,02	0.000,685	0.000,952	0.008,867	0.01,2325
180	0.992,318	0.989,338	0.904,96	0.870,393	0.003,349	0.004,655	0.043,37	0.060,285
240	0.881,485	0.839,169	0.195,241	0.103,252	0.054,785	0.076,151	0.709,428	0.986,101
300	0.350,289	0.232,678	1.26E-06	6.31E-09	0.455,574	0.633,245	5.899,379	8.200,106
360	0.000,433	2.11E-05	2.75E-44	2.82E-61	3.363,977	4.675,91	43.561,28	60.549,95

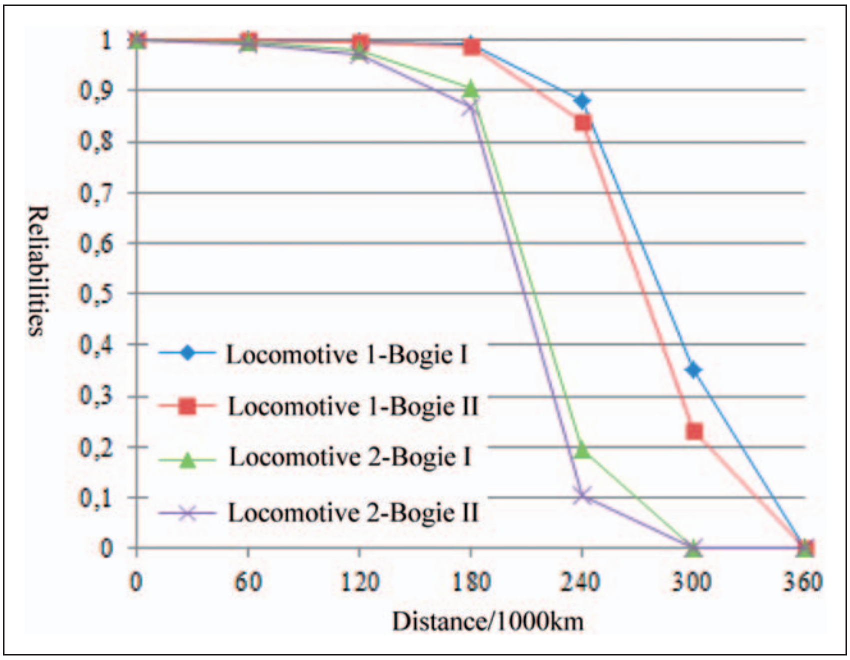

Figure 4. Plot of the reliabilities for locomotive 1 and locomotive 2.

240,000 km, using irregular intervals could be another choice for some specified case studies.)

Once we settled on a model, the task became to infer the desired parameters. For this purpose, we resorted to Monte Carlo integration which essentially draws samples from the required distribution, and then forms sample averages to approximate expectations. The recent proliferation of MCMC approaches has led to the use of the Bayesian inference in a wide variety of fields, including behavioural science, finance, human health, process control, ecological risk assessment, and risk assessment of engineered systems. MCMC draws these samples by running a cleverly constructed Markov chain for a long time. There are many ways of constructing these chains. One of the simplest MCMC sampling algorithms found in the Bayesian computational literature is the Gibbs sampler. The literature concerning the MCMC method using a Gibbs sampler is too vast to be listed here. In this paper, the method is used to integrate over the posterior distribution of model parameters given the data, this to make inference for the desired model parameters.

For convenience, we let $\lambda_k = \exp(b_k)$, and the following vague prior distributions were adopted:

- gamma frailty prior: $\omega_i \sim Ga(\kappa^{-1}, \kappa^{-1})$;
- normal prior distribution: $b_k \sim N(b_{k-1}, \kappa)$;
- normal prior distribution: $b_1 \sim N(0, \kappa)$;
- gamma prior distribution: $\kappa \sim Ga\ (0.0001, 0.0001)$;
- normal prior distribution: $\beta_0 \sim N(0.0, 0.001)$;
- normal prior distribution: $\beta_1 \sim N(0.0, 0.001)$.

At this point, the MCMC calculations were implemented with the software WinBUGS.[38] The initial values were generated by WinBUGS in our case and a burn-in of 10,001 samples was used, with an additional 10,000 Gibbs samples.

Results

Following the convergence diagnostics (including checking dynamic traces in Markov chains, time series, and comparing the Monte Carlo (MC) error with standard deviation (SD); see Lunn et al.[38]), we consider the following posterior distribution summaries (Table 3). Statistics summaries include the parameters' posterior distribution mean, SD, MC error, and the 95% highest posterior distribution density (HPD) interval.

In Table 3, $\beta_1 > 0$ means that the wheels mounted on the first bogie (as $x = 1$) have a shorter lifetime than those on the second bogie (as $x = 2$). However, the influence may be possibly reduced as more data are obtained in the future, because the 95% HPD interval includes the zero point. Because κ is likely to be > 0.5, there is probably a positive relationship between the wheels mounted on the same locomotive; in addition, the heterogeneity among the locomotives is significant. Meanwhile, $\omega_1 < 1$ suggests that the predicted lifetimes for those wheels mounted on the first locomotive are longer than if the frailties are not considered; meanwhile, ω_2 is likely to be > 1 which would indicate the wheels mounted on the second locomotive have shorter lifetime than if the frailties are not considered.

Baseline hazard rate statistics based on the above results are shown in Table 4 and Figure 3. The red arrows marked in Figure 3 underline that the wheels' baseline hazard rate increases dramatically after the fourth piecewise interval. One possible reason for the increase is that, in our case study, there is no failure data until the fourth interval.

By considering the random effects resulting from the natural variability (explained by covariates) and from the unobserved random effects within the same group (explained by frailties), we can determine other reliability characteristics of the lifetime distribution. The statistics on reliability $R(t)$ and cumulative hazard rate $\Lambda(t)$ for the two wheels mounted on different bogies are listed in Table 5. Figure 4 and Figure 5 show the reliabilities and cumulative hazards for locomotive 1 and locomotive 2, respectively. In addition, for these locomotives, the wheels mounted in the first bogie ($x = 1$) have lower reliability and a higher cumulative hazard rate than those mounted in the second one ($x = 2$). In addition, Figure 4 and Figure 5 show that the wheels have a high risk of failure after they have run for a certain distance. For example, the reliability declines sharply at the fourth piecewise interval, and at the fifth piecewise interval, the cumulative hazard increases dramatically.

Discussion

The presented results can be applied to maintenance optimisation procedures, including lifetime prediction and replacement optimisation, preventive maintenance optimisation, and reprofiling optimisation.

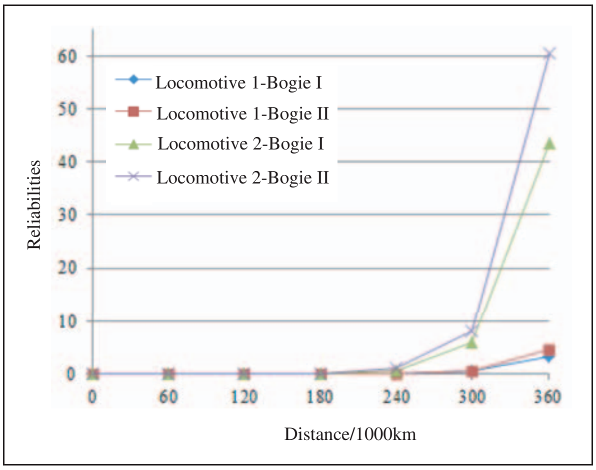

Figure 5. Plot of the cumulative hazard for locomotive 1 and locomotive 2.

First, determining the distribution of reliability characteristics over the wheels' lifetime could be used to optimise replacement strategies. The results could also support related predictions for spares inventory.

Second, the dramatically increasing points (Figures 3 to5) appearing in the fourth and fifth piecewise interval (from 180,000 to 300,000 km) indicate that after running about 180,000 km, a locomotive's wheel has a high risk of failure. To understand the possible reasons for this phenomenon, we held on-site discussions with operators and maintenance engineers. Using the knowledge gained from their experience, they suggested that the observed behaviour could be the result of rolling contact fatigue problems starting in the fifth interval (after 240,000 kilometres). Therefore, special attention should be paid if the wheels have run longer than these points.

Third, the wheels installed on the first bogie should be paid more attention during maintenance. Especially when the wheels are reprofiled, they should be checked starting with the first bogie to avoid duplication of effort. Note that in the studied company, the first checked wheel could belong in the second bogie. After the second checked wheel is lathed or reprofiled, if the diameter is smaller than predicted, the first checked wheel might need to be lathed or reprofiled again. Therefore, starting with the wheel installed on the first bogie could improve maintenance effectiveness.

Last, but by no means least, the frailties between locomotives could be caused by different operating environments (e.g. climate, topography and track geometry), configuration of the suspension, status of the bogies or spring systems, operation speeds and applied loads. Specific operating conditions should be considered when designing maintenance strategies because even if the locomotives and wheel types are the same, the lifetimes and operating performance could differ.

Conclusions

This paper proposes a Bayesian semi-parametric framework to analyse a locomotive wheel's reliability using degradation data. The piecewise constant hazard rate is used to analyse the distribution of a wheel's lifetime. The gamma shared frailties $\boldsymbol{\omega}_i$ are used to explore the influence of unobserved covariates within the same locomotive. By introducing covariate $\mathbf{x}_i$'s linear function $\mathbf{x}_i^{'}\boldsymbol{\beta}$, the influence of the bogie on which a wheel is installed can be taken into account. The proposed framework can deal with small and incomplete datasets and simultaneously consider the influence of different covariates. The MCMC technique is used to integrate high-dimensional probability distributions to make inferences and predictions about model parameters.

The results of the case study suggest that a polynomial degradation path for the wheels is more appropriate than a linear degradation path. The lifetimes of wheels probably differ according to where they are installed (on which bogie they are mounted) on the locomotive. Wheels installed on the second bogie have longer lifetimes than the ones on the first bogie. We have an educated guess that the differences could be influenced by the real running situation (e.g. topography) and the locomotive's centre of gravity. The gamma frailties help with exploring the unobserved covariates and thus improve the model's accuracy. Results also indicate a close positive relationship between wheels mounted on the same locomotive; the heterogeneity between locomotives is also significant. We can determine the wheel's reliability characteristics, including the baseline hazard rate $\lambda(t)$, reliability $R(t)$ and cumulative hazard rate $\Lambda(t)$. As shown in Figures 3 to 5, wheel reliability probably declines sharply at the fourth piecewise interval, while at the fifth piecewise interval, the cumulative hazard increases dramatically. The results allow us to evaluate and optimise wheel replacement and maintenance strategies (including the reprofiling interval, inspection interval, lubrication interval, depth and optimal sequence of reprofiling, etc.).

Finally, the approach discussed in this paper can also be applied to cargo train wheels or passenger train wheels.

We suggest the following additional research:

1. The covariates considered in this paper are limited to the installed positions of the locomotive wheels; more covariates must be considered. To this end, we will study such factors as operating environment (e.g. climate, topography and track geometry), configuration of the suspension, status of the bogies and the spring systems, operation speeds and the applied loads, etc. By considering more and more factors such as those listed, we expect to improve the accuracy of the model step by step. The goal will be to find out the key influence factors which will influence the lifetime of the wheels the most and explore their impact.
2. We have taken both the first and the second bogie in one integrated model. Later, taking first and second bogie data as two different distributions and comparing these should be studied.
3. The causes of the differences between bogies are not explored in this paper, which are addressed by vehicle dynamics and work in this field. They should be studied further.
4. We have chosen vague prior distributions for the case study. Other prior distributions, including both informative and non-informative prior distributions, should be studied.
5. Although the diagnostic of the MCMC method show the gamma shared frailty could be one choice, however, other frailty models should be studied.
6. In subsequent research, we plan to consider using our results to optimise maintenance strategies and the related Life Cycle Cost problem considering maintenance costs, particularly with respect to different maintenance inspection levels and inspection periods (long, medium and short term).

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