Flexural Strengthening with NSM

A design example

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Abstract

This report presents a practical example for transverse flexural strengthening of RC trough bridge decks. In order to increase the maximum axle loads on a trough bridge of type B2447-2 from 25 to 30 ton, two NSM bars of type StoFRP Bar IM10C is needed for every theoretical strip of 1 meter.

Table of content

1. General

Many of the existing railway bridges were designed for trains with characteristic axle loads of 25 ton. This report presents the calculations required to upgrade the flexural resistance of a concrete bridge by introducing Near Surface Mounted (NSM) bars in the bottom surface of the bridge deck. A fictional transverse strip with a width of 1.0 m is studied and the cross sectional parameters are illustrated in figure 1.1 and 1.2. The bridge is a short span trough bridge, denoted B2447-2 with a free span of 4.0 m. The concrete deck contains both tensile and compressive reinforcement, but the latter is neglected due to a relatively small ratio.

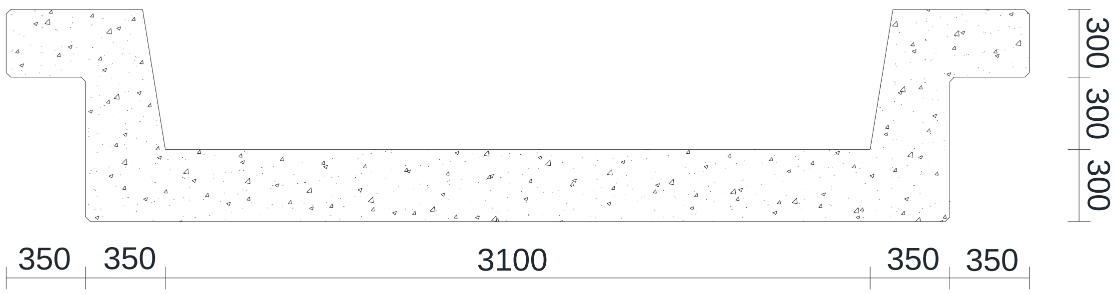

Figure 1.1. Cross section and dimensions of the B2447-2 trough bridge.

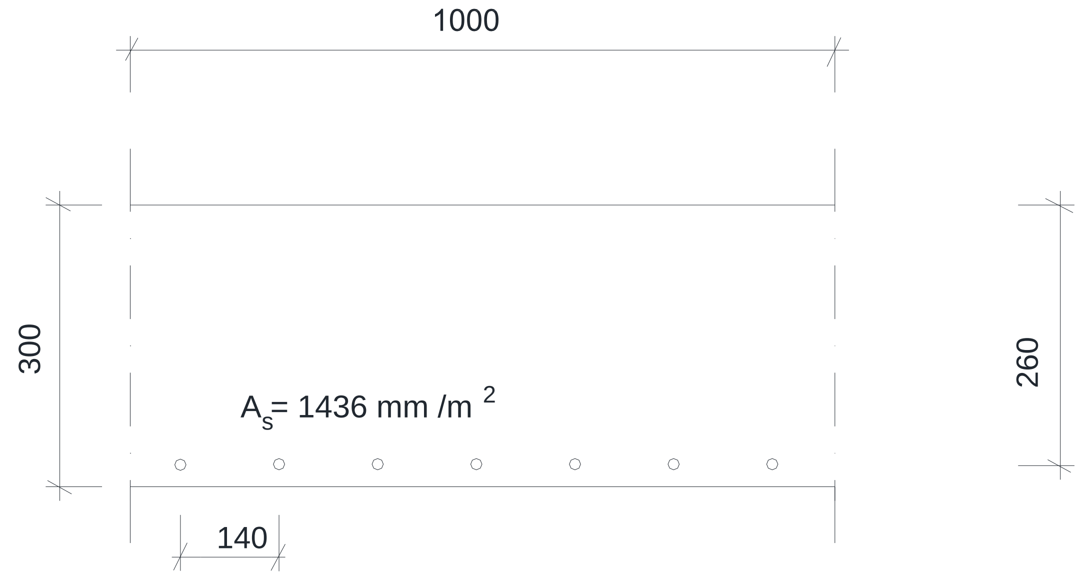

Figure 1.2. Cross section and dimensions of a transverse mid-strip for design of the bridge deck.

The strip is assumed to be simply supported between the midpoints of the main girders as seen in figure 3.1.

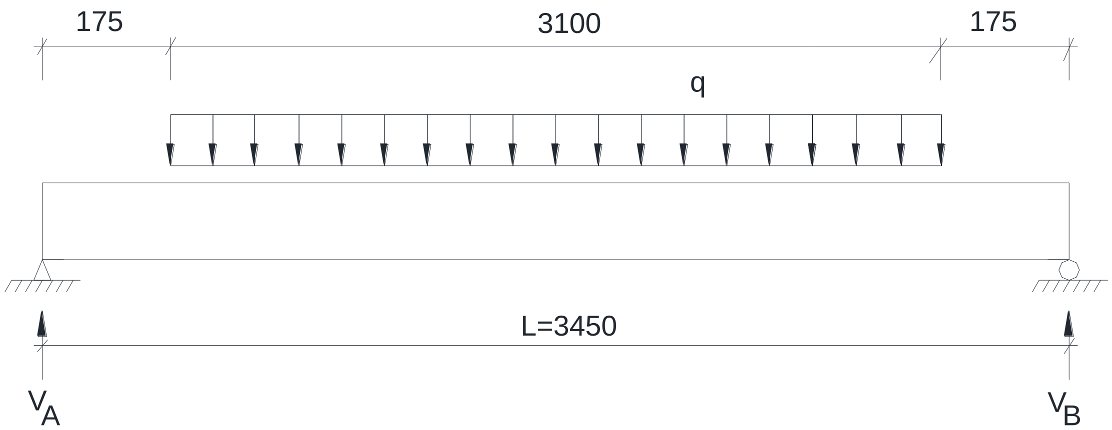

Figure 1.3. The bridge deck is assumed to be simply supported between the main girders.

2. Material data

Material properties have not been tested and the original recommendations for characteristic values along with design values according to prevailing standards are presented in the tables below:

Concrete: K400

Compressive strength	f_{cck}	$37.5\ MPa$
	f_{ccd}	$20.8\ MPa$
Tensile strength	f_{ctk}	$2.5\ MPa$
	f_{ctd}	$1.4\ MPa$
Young's Modulus	E_c	$34.0\ GPa$
Creep	φ_c	$2.0\ \ (dead\ loads)$ $0\ \ (live\ loads)$
Effective modulus of elasticity	$E_{ce} = \frac{E_{ck}}{1+\varphi_e}$	$\frac{34}{1+2} = 11.3\ (dead\ loads)$ $\frac{34}{1+0} = 34.0\ \ (live\ loads)$

Reinforcement: Ks40, Ø16 mm

Tensile strength	f_{yk}	$400\ MPa$
	f_{yd}	$290\ MPa$
Young's Modulus	E_s	$200\ GPa$
Concrete cover	c	$30\ mm$
Scale factor	$\propto_s = \frac{E_s}{E_{ce}}$	$\frac{200}{11.3} = 17.7 \quad (dead\ loads)$ $\frac{200}{34} = 5.9 \quad (live\ loads)$

NSM: StoFRP Bar IM10C

Elongation	ε_{fu}	1.57 %
	ε_{fd}	1.09 %
Young's Modulus	E_f	$210\ GPa$
	E_{fd}	$146\ GPa$
Area	A_f	$10 \cdot 10\ mm$

3. Design principles

The principles for flexural strengthening are shown in figure 3.1, where the different strains are illustrated. An important aspect to consider during the design is the existence of possible strain fields due to for example the dead loads. In figure 3.1 b) this is shown schematically, where the dashed line represents the initial strain in the cross section, and the solid line illustrates the strain in the strengthened cross section during live loading.

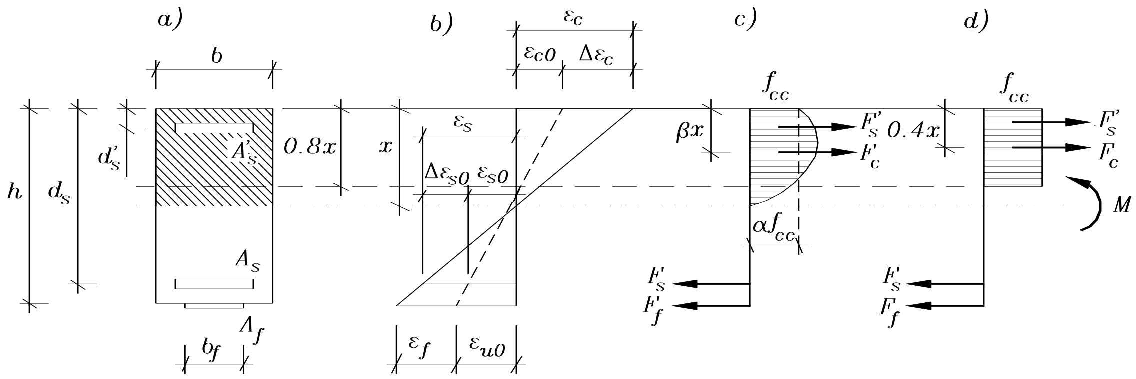

Figure 3.1. Principle for flexural strengthening

The design is initiated by calculating the actual strain levels in the cross section, where the influence of creep in the concrete is considered by introducing a reduced modulus of elasticity:

$$E_{ce} = \frac{E_{ck}}{1 + \varphi_e} \quad (3:1)$$

The concrete remains uncracked as long as the tensile stress has not exceeded $f_{ctm,fl}$:

$$f_{ctm,fl} = max\left\{\left(1.6 - \frac{h}{1000}\right) f_{ctm};\ f_{ctm}\right\} \tag{3:2}$$

This is controlled by calculating the concrete tensile stress in the bottom part of the studied section:

$$\sigma_c = \frac{M_0}{I_c + \alpha_s I_s} y = \frac{M_0}{I_1} y \tag{3:3}$$

where I_c, and I_s is the moment of inertia for the concrete and steel, respectively. The distance from the centre of gravity to the top of the studied cross section can generally be calculated as:

$$y_0 = \frac{A_c y_{tp,c} + (\alpha_s - 1) A_s d_s}{A_c + (\alpha_s - 1) A_s} \tag{3:4}$$

For a rectangular cross section, equation (3:4) can be written as

$$y_0 = \frac{bh\frac{h}{2} + (\alpha_s - 1) A_s d_s}{bh + (\alpha_s - 1) A_s} \tag{3:5}$$

The ideal moment of inertia for a rectangular cross section, with the width *b* and height *h* can then be calculated as follows:

$$I_1 = I_c + \alpha_s I_s = \frac{bh^3}{12} + bh\left(y_0 - \frac{h}{2}\right)^2 + (\alpha_s - 1) A_s (d_s - y_0)^2 + (\alpha_s - 1) A'_s (d'_s - y_0)^2 \tag{3:6}$$

The next step is to calculate the stresses in the cross section. The highest stress magnitudes are found at top, σ_{ct}, and bottom, σ_{cb}, of the studied section:

$$\sigma_{ct} = \frac{M_0}{I_c + \alpha_s I_s} y_0 = \frac{M_0}{I_1} y_0 \tag{3:7}$$

$$\sigma_{cb} = \frac{M_0}{I_c + \alpha_s I_s}(h - y_0) = \frac{M_0}{I_1}(h - y_0) \tag{3:8}$$

The stress in the tensile reinforcement is:

$$\sigma_s = \propto_s \frac{M_0}{I_c + \alpha_s I_s}(d_s - y_0) = \propto_s \frac{M_0}{I_1}(d_s - y_0) \tag{3:9}$$

The strains in the uncracked section can then be calculated:

$$\varepsilon_{ct} = \frac{\sigma_{ct}}{E_{ce}} \tag{3:10}$$

$$\varepsilon_{cb} = \frac{\sigma_{cb}}{E_{ce}} \tag{3:11}$$

In stadium II, the concrete will be cracked, $\sigma_{cuk} > f_{ctm,fl}$, and in the design calculations, the concrete is considered to only resist compressive stresses. The expression for the moment of inertia becomes different, see figure 3.2.

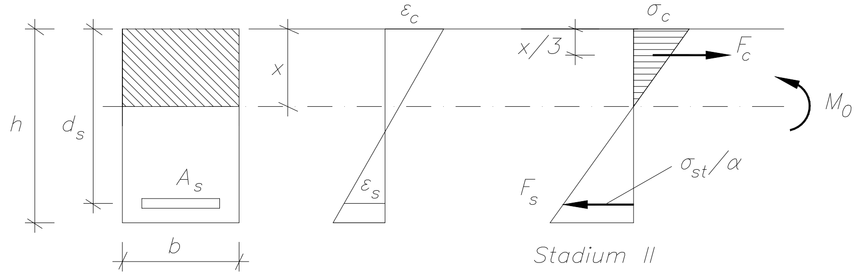

Figure 3.2 The cross section is in stadium II

The moment of inertia for a rectangular cross section in stadium II can be calculated as:

$$I_2 = I_c + \alpha_s I_s = \frac{bx^3}{12} + bx\left(\frac{x}{2}\right)^2 + (\alpha_s - 1)A'_s(x - d'_s)^2$$
$$+\alpha_s A_s(d_s - x)^2 \qquad (3:12)$$

where the distance from the top of the section to the neutral axis can be calculated as:

$$bx\frac{x}{2} + (\alpha_s - 1)A'_s(x - d'_s) = \alpha_s A_s(d_s - x) \Rightarrow$$
$$x^2 + \frac{2}{b}[(\alpha_s - 1)A'_s + \alpha_s A_s] - \frac{2}{b}[(\alpha_s - 1)A'_s d'_s + \alpha_s A_s d_s] \qquad (3:13)$$

The stresses and strains over the section can then be calculated using equation's *(3:10-11)*. A first estimation for the necessary area of the fibre composite material to increase the bending moment can be made with:

$$M = A'_s f'_s(\beta x - d'_s) + A_s f_s(d_s - \beta x) + \varepsilon_f E_f A_f(h - \beta x) \qquad (3:14)$$

However, if the type of failure that can occur is assumed to be failure in the composite material without yielding in the compressive reinforcement, the bending capacity can then be expressed as, see also Täljsten, 2006:

$$M = \frac{x - d'_s}{h - x}(\varepsilon_{fu} + \varepsilon_{u0})A'_s E_s(\beta x - d'_s) + A_s f_s(d_s - \beta x) + \varepsilon_f E_f A_f(h - \beta x) \tag{3:15}$$

a horizontal equilibrium equation for the section in figure 3.1 gives:

$$\alpha f_{cc} b x + \frac{x - d'_s}{h - x}(\varepsilon_{fu} + \varepsilon_{u0})A'_s E_s = A_s f_s + \varepsilon_f E_f A_f \tag{3:16}$$

where x can be solved with an equation of the second degree:

$$C_1 x^2 + C_2 x + C_3 = 0 \tag{3:17}$$

Where

$$\left.\begin{aligned} C_1 &= \alpha f_{cc} b \\ C_2 &= -\alpha f_{cc} b - (\varepsilon_{fu} + \varepsilon_{u0})A'_s E_s - A_s f_s - \varepsilon_f E_f A_f \\ C_3 &= (\varepsilon_{fu} + \varepsilon_{u0})A'_s E_s d'_s + (A_s f_s + \varepsilon_f E_f A_f)h \end{aligned}\right\} \tag{3:18}$$

4. Actions

4.1. *Permanent actions*

The bending moment due to permanent actions in a 1m wide transverse strip is first determined. Two types of permanent actions are considered in this example; the self-weight of the concrete and the additional weight of the ballast. The modelling assumptions for the bending moment due to the ballast shown in figure 4.1.

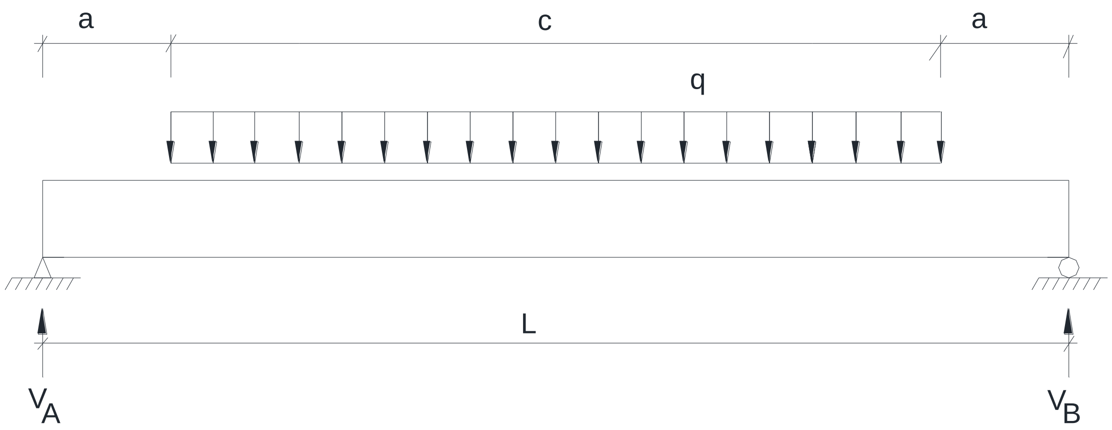

Figure 4.1 Model for calculating bending moment due to ballast action.

While the ballast acts as a distributed load on a simply supported beam, the self-weight of the concrete is assumed to act as a distributed load on a clamped beam, as seen in figure 4.2.

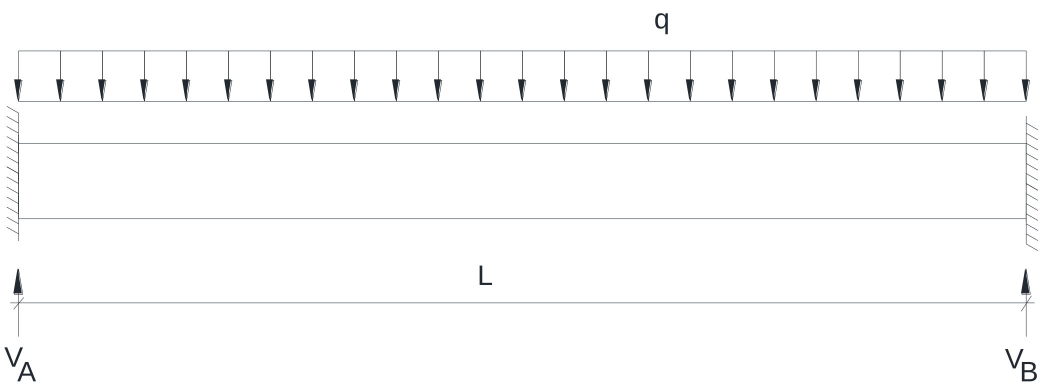

Figure 4.2 Model for calculating bending moment due to concrete action.

The permanent loads, q, in a 1m wide strip are:

$$q_{concrete} = \rho_{concrete} \cdot h = 24 \cdot 0.30 = 7.2\ kN/m$$

$$q_{ballast} = \rho_{ballast} \cdot h = 20 \cdot 0.6 = 12.0\ kN/m$$

The maximum moments in the midspan strip due to permanent actions are hence calculated as:

$$M_{concrete} = \frac{q_{concrete} \cdot L^2}{16} = \frac{7.2 \cdot 3.45^2}{16} = 5.4\ kNm/m$$

$$M_{ballast} = V_A x - \frac{q_{ballast}}{2}(x-a)^2 = 18.6 \cdot 1.725 - \frac{12}{2}(1.725 - 0.175)^2$$

$$= 17.7\ kNm/m$$

The total maximum moment from the permanent actions is thereby:

$$M_0 = 5.4 + 17.7 = 23.1\ kNm/m$$

4.2. *Variable actions*

The variable actions are in this case represented by the train load. The axle loads from the train are distributed by the rail, the sleepers and the ballast, and the axle load can therefore be translated into one uniformly distributed load acting on the bridge deck. Since the axle distance typically is $s = 1.6$ m and the transverse length of the loaded part of the bridge deck is $L = 3.1$ m, the variable load from a typical train with the axle load $P_{axle} = 250\ kN$, can be represented by a distributed load q_{train}:

$$q_{train} = \frac{P_{axle}}{s \cdot L} = \frac{250 \cdot 10^3}{1.6 \cdot 3.1} = 50.4\ kN/m$$

The moment contribution from variable action is modeled in a similar manner as the ballast, see figure 4.1. The maximum moment in the midspan strip is thus:

$$M_{train} = V_A x - \frac{q_{train}}{2}(x - a)^2 = 78.12 \cdot 1.725 - \frac{50.4}{2}(1.725 - 0.175)^2$$

$$= 74.2\ kNm/m$$

The design value for the moment is the total sum of permanent and variable loads:

$$M_{Ed} = M_0 + M_{train} = 23.1 + 74.2 = 97.3\ kNm/m$$

5. Flexural capacity

5.1. Initial capacity

The first step is to investigate whether the bridge is cracked and calculate the distance to the neutral axis, y_0:

$$y_0 = \frac{A_c y_{tp,c} + (\alpha_s - 1)A_s d_s + (\alpha_s - 1)A'_s d'_s}{A_c + (\alpha_s - 1)A_s + (\alpha_s - 1)A'_s}$$

$$= \frac{1.0 \cdot 0.30 \cdot 0.15 + (17.7 - 1)1436 \cdot 10^{-6} \cdot 0.26}{1 \cdot 0.3 + (17.7 - 1)1436 \cdot 10^{-6}} = 0158\ m$$

The moment of inertia for the uncracked section is:

$$I_1 = I_c + \alpha_s I_s$$

$$= \frac{bh^3}{12} + bh\left(y_0 - \frac{h}{2}\right)^2 + (\alpha_s - 1)A_s(d_s - y_0)^2 + (\alpha_s - 1)A'_s(d'_s - y_0)^2$$

$$= \frac{1 \cdot 0.3^3}{12} + 1 \cdot 0.3\left(0.158 - \frac{0.3}{2}\right)^2 + (17.7 - 1)1436 \cdot 10^{-6}(0.26 - 0.158)^2$$

$$+(17.7 - 1)314 \cdot 10^{-6}(0.04 - 0.158)^2 = 2.52 \cdot 10^{-3}\ m^4$$

The initial stresses and strains can thereafter be determined:

$$\sigma_{ct} = \frac{\mathrm{M}_0}{I_1} y_0 = \frac{23.1 \cdot 10^3}{2.52 \cdot 10^{-3}} \cdot 0.158 = 1.4\ MPa$$

$$\sigma_{cb} = \frac{\mathrm{M}_0}{I_1}(h - y_0) = \frac{23.1 \cdot 10^3}{2.52 \cdot 10^{-3}} \cdot (0.3 - 0.158) = 1.3\ MPa$$

$$\sigma'_s = (\alpha_s - 1)\frac{\mathrm{M}_0}{I_1}(d'_s - y_0) = 16.7 \cdot \frac{23.1 \cdot 10^3}{2.52 \cdot 10^{-3}} \cdot (0.04 - 0.158) = -18.1\ MPa$$

$$\sigma_s = (\alpha_s - 1)\frac{M_0}{I_1}(d_s - y_0) = 16.7 \cdot \frac{23.1 \cdot 10^3}{2.52 \cdot 10^{-3}} \cdot (0.26 - 0.158) = 15.6\ MPa$$

$$\varepsilon_{ct} = \frac{\sigma_{ct}}{E_{ce}} = \frac{1.4 \cdot 10^6}{11.3 \cdot 10^9} = 0.124 \cdot 10^{-3}$$

$$\varepsilon_{cb} = \frac{\sigma_{cb}}{E_{ce}} = \frac{1.3 \cdot 10^6}{11.3 \cdot 10^9} = 0.115 \cdot 10^{-3}$$

$$\varepsilon'_s = \frac{\sigma'_s}{E_s} = \frac{18.1 \cdot 10^6}{200 \cdot 10^9} = 0.091 \cdot 10^{-3}$$

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{15.6 \cdot 10^6}{200 \cdot 10^9} = 0.078 \cdot 10^{-3}$$

The bridge can thereby be considered as uncracked for the permanent actions. The ultimate capacity of the unstrengthened bridge can be determined by the following equilibrium equations where the compressive reinforcement is neglected. First the neutral layer is determined:

$$\alpha f_{cc} b x = A_s f_s$$

$$x = \frac{A_s f_{sd}}{\alpha f_{ccd} b} = \frac{1436 \cdot 10^{-6} \cdot 290 \cdot 10^6}{0.8 \cdot 20.8 \cdot 10^6 \cdot 1} = 0.025\ m$$

The flexural capacity of the unstrengthened bridge deck is:

$$M_{Rd} = A_s f_s (d_s - \beta x) = 1436 \cdot 10^{-6} \cdot 290 \cdot 10^6 \cdot (0.26 - 0.4 \cdot 0.025)$$

$$= 104\ kNm/m$$

The unstrengthened bridge deck in this example hence has a capacity to carry trains with axle loads up to 250 kN, with respect to moment.

$M_{Rd} > M_{Ed} \quad \Rightarrow \quad OK!$

5.2. Strengthening with NSM

By increasing the axle loads from 250 to 300 kN, the bridge deck requires strengthening as shown below:

$$q_{train} = \frac{P_{axle}}{s \cdot L} = \frac{300 \cdot 10^3}{1.6 \cdot 3.1} = 60.5\ kN/m$$

$$M_{train} = V_A x - \frac{q_{train}}{2}(x-a)^2 = 93.775 \cdot 1.725 - \frac{60.5}{2}(1.725 - 0.175)^2$$

$$= 89.1\ kNm/m$$

$$M_{Ed} = M_0 + M_{train} = 23.1 + 89.1 = 112.2\ kNm/m$$

$M_{Rd} < M_{Ed} \quad \Rightarrow \quad NOT\ OK!$

The strengthening in this example consists of NSM CFRP bars installed in the bottom surface of the deck.

The first step is to make a rough estimation of the required amount of FRP:

$$A_f = \frac{\frac{M_{Ed}}{0.9} - A_s f_s d}{\varepsilon_{fd} E_{fd} h} = \frac{\frac{115 \cdot 10^3}{0.9} - 1436 \cdot 10^{-6} \cdot 290 \cdot 10^6 \cdot 0.26}{10.9 \cdot 10^{-3} \cdot 146 \cdot 10^9 \cdot 0.3} = 41\ mm^2$$

Approximately 1 StoFRP IM10C bar is needed for the strengthening, giving a total fibre area of 100 mm^2/m.

The distance to the neutral axis of the strengthened bridge deck is determined by solving the horizontal equilibrium equation:

$$\alpha f_{cc} b x = A_s f_s + \varepsilon_f E_f A_f$$

$$x = \frac{A_s f_s + \varepsilon_f E_f A_f}{\alpha f_{cc} b} =$$

$$\frac{1436 \cdot 10^{-6} \cdot 290 \cdot 10^{6} + 10.9 \cdot 10^{-3} \cdot 146 \cdot 10^{9} \cdot 100 \cdot 10^{-6}}{0.8 \cdot 20.8 \cdot 10^{6} \cdot 1} = 35\ mm$$

The moment resistance is:

$$M_{Rd} = A_s f_s (d_s - \beta x) + \varepsilon_f E_f A_f (h - \beta x) =$$

$$1436 \cdot 10^{-6} \cdot 290 \cdot 10^{6} (0.26 - 0.4 \cdot 0.035) +$$

$$10.9 \cdot 10^{-3} \cdot 146 \cdot 10^{9} \cdot 100 \cdot 10^{-6} (0.3 - 0.4 \cdot 0.035) = 148\ kNm/m$$

The resistance is thereby significantly higher than what is required and the bridge deck is now capable of carrying trains with axle loads up to 420 kN. EN1990 however recommends using a partial factor of 1.5 for variable loads and hence a second NSM bar will be required, since the actual axle load is reduced to 280 kN.

The distance to the neutral axis when 2 NSM bars are used:

$$\alpha f_{cc} b x = A_s f_s + \varepsilon_f E_f A_f$$

$$x = \frac{A_s f_s + \varepsilon_f E_f A_f}{\alpha f_{cc} b} =$$

$$\frac{1436 \cdot 10^{-6} \cdot 290 \cdot 10^{6} + 10.9 \cdot 10^{-3} \cdot 146 \cdot 10^{9} \cdot 200 \cdot 10^{-6}}{0.8 \cdot 20.8 \cdot 10^{6} \cdot 1} = 44\ mm$$

The moment resistance is increased to:

$$M_{Rd} = A_s f_s (d_s - \beta x) + \varepsilon_f E_f A_f (h - \beta x) =$$

$$1436 \cdot 10^{-6} \cdot 290 \cdot 10^{6} (0.26 - 0.4 \cdot 0.044) +$$

$$10.9 \cdot 10^{-3} \cdot 146 \cdot 10^{9} \cdot 200 \cdot 10^{-6} (0.3 - 0.4 \cdot 0.044) = 191\ kNm/m$$

A total moment resistance of $M_{Rd} = 191\ kNm/m$ corresponds to a maximum variable load of 114 kN/m, and considering the partial factor this corresponds to a maximum axle load of 377 kN.